

Admny

Midterm tomorrow

Hw today (or Monday (or later...))

be careful! it missing @ the
end, I might not get to grading

it

also, take-home will be stricter

getting closer on grading Hw}

No OT tomorrow

Today: Review

Next week: semidirect products

Problems from Lang Algebra Ch. 1

S. Let $H \leq G_1 \times G_2$.

"11 hours at's drumming" c.f. Wikipedia
for a proof w/ coset bashing

Consider $\pi_i: G_1 \times G_2 \rightarrow G_i$, suppose $\pi_i|_H$ onto.

Let $N_1 = \ker(\pi_2|_H)$, $N_2 = \ker(\pi_1|_H)$.

i. View $N_i \trianglelefteq G_i$

ii. $G_1/N_1 \cong G_2/N_2$

iii. The image of H in $\frac{G_1}{N_1} \times \frac{G_2}{N_2}$ is the

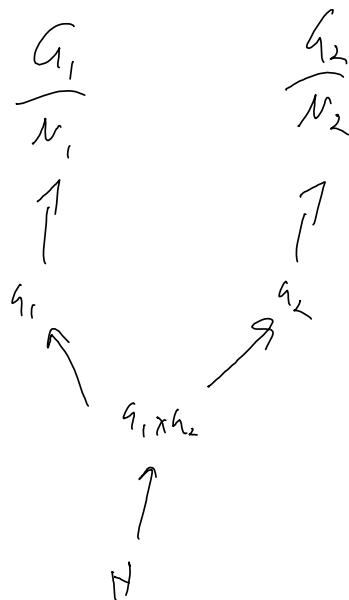
graph of the iso morphism from (ii).

Pr. i. $\ker(\pi_1) = e \times G_2$, $\ker(\pi_2) = G_1 \times e$
 $\uparrow$ $\uparrow$
 $\ker(\pi_1|_H) = N_2$ $\ker(\pi_2|_H) = N_1$

$N_i \trianglelefteq H$, so their "intersections" w/ the G_i are normal in G_i .

ii. How to find a map???

No dup.



π_2 first in turn, π_2 induces $\frac{H}{N_2} \xrightarrow{\sim} g_1$

π_1 induces $\frac{H}{N_1} \xrightarrow{\sim} g_2$

so we expect $\frac{g_1}{N_1} \cong \frac{H}{N_1 N_2} \cong \frac{g_2}{N_2}$

Indeed, consider $\frac{H}{N_2} \xrightarrow[\sim]{\pi_1} g_1 \longrightarrow g_1/N_1$

what's the kernel?

$$\begin{array}{ccc} H & \xrightarrow{\pi_1} & g_1 \\ \downarrow & \nearrow \pi_1 & \\ H/N_2 & & \end{array} \quad \frac{\pi_1^{-1}[N_1]}{N_2}$$

what's $\pi_1^{-1}(N_1)$?

Lemma, $f: A \rightarrow B$, $C \subseteq B$,

suppose $D \subseteq A$ s.t. $f[D] = C$

Then $f^{-1}[C] = D \cup \ker(f)$

Pr. " $\supseteq$ " clear
 $f^{-1}[C] \longrightarrow C$ has kernel $\ker(f)$,
 " $\subseteq$ " as $\ker(f) \subseteq C$

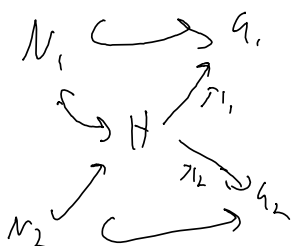
$$\frac{f^{-1}[C]}{\ker(f)} \cong C \text{ via } f$$

$$\text{OTOH, } \frac{D \cup \ker(f)}{\ker(f)} \cong \frac{D}{D \cap \ker(f)}$$

$D \longrightarrow C$ has kernel $D \cap \ker(f)$

so this is iso to C

Thus by correspondence (check details!), $\frac{D \cup \ker(f)}{\ker(f)} \cong f^{-1}[C]$ $\square$



Apply lemma to $\pi_1: H \rightarrow G_1$
 $C = N_1 \times \{e\}$

$D = N_1$

to get $\pi_1^{-1}[C] = N_1 \cup N_2$

$$\text{so } \ker(H \rightarrow g_1/N_1) \text{ is } \frac{N_1 N_2}{N_1}$$

$$\text{same for } H \rightarrow g_2/N_2$$

$$\frac{H/N_1}{N_1 N_2 / N_1} \xleftarrow[\text{"id"}]{\sim} \frac{H}{N_1 N_2} \xrightarrow[\text{"id"}]{\sim} \frac{H/N_2}{N_1 N_2 / N_2}$$

$$\begin{array}{ccc} \sim \downarrow \text{"}\pi_1\text{"} & & \text{"}\pi_2\text{"} \downarrow \sim \\ \frac{g_1}{N_1} & & \frac{g_2}{N_2} \end{array}$$

iii.

$$\begin{array}{ccccc} & & \sim & & \frac{g_1}{N_1} \\ & \nearrow \text{"}\pi_1\text{"} & & \nearrow & \\ \frac{H}{N_1 N_2} & \leftarrow H & \rightarrow & \frac{g_1}{N_1} & \times \frac{g_2}{N_2} \quad | \quad \sim \\ & \searrow & & \searrow & \\ & & \sim & & \frac{g_2}{N_2} \end{array}$$

"π₂"

image is $(h_1, N_1, h_2, N_2) \mid (h_1, h_2) \in H_1 \times H_2$
 graph is $(g_1, N_1, g_2, N_2) \mid g_1, N_1 \mapsto g_2, N_2$ via the i.o. Then, both come from $H/N_1 N_2$ $\square$