

Admin

- HW3 due today
- I'll be faster grading after this week
- I'll be strict w/ the take home deadline; Th wk 10 11:59pm

Cryptography

Goal. Communicate securely over insecure channels.

A good encryption standard should ensure

- Secrecy - no one can read it but those who are allowed
- unmodifiability - it cannot be modified in transit
- authentic endpoints - Sender and receiver can be assured of each other's identity (possibly pseudonymous)

We focus on Secrecy here.

Naïve ciphers:

- Fix a permutation of the alphabet and agree on it beforehand. Scramble messages thus. This is the

Caesar cipher.

Faults: - Easy to decode (repeated letters)

- once decoded, all previous messages are decodable
- must agree beforehand!

- Is it possible for strangers to communicate secretly?

- Pig Latin

Caesar cipher is symmetric: both parties use the same "key".

In the 70's Diffie and Hellman revolutionized cryptography by a method for two strangers to agree on a public key.

Later on Rivest, Shamir, and Adleman created a "public key" cryptosystem (RSA) which allows messages to be transmitted securely without any prior agreement or shared key.

Rmk. Symmetric encryption is often faster for 2 peers seeking to communicate, so nowadays your browser will use a key-exchange alg. Diffie-Hellman to generate a shared secret, then use a

Diffie-Hellman

Lemma, $(\mathbb{Z}/p\mathbb{Z})^*$ cyclic

pf. $\exists f$ s.t. $x^{d-1} = 1$ $d \leq p-1$ *

A	B	M
g	g	g
g^a	g^b	g^a
g^{ab}	g^{ab}	g^b

$$\log_g : \langle g \rangle \longrightarrow \mathbb{Z}/(p-1)\mathbb{Z}$$

$$g \longmapsto 1$$

Math is declarative } S.I.C.P.
CS is imperative }

RSA

Let p, q distinct primes.

$n = pq$ $\nearrow$ the exponent of $(\mathbb{Z}/n\mathbb{Z})^\times$

Compute $\text{lcm}(p-1, q-1) = k$

Pick e s.t. $\gcd(e, k) = 1$
 $1 < e < k$

Compute $d \equiv e^{-1} \pmod{k}$

public key: (n, e) $\text{Prv. They don't know } p, q!$

private key: d

Say your message m is encoded as an integer (ASCII, unicode, etc) s.t. $0 \leq m < n$.

The cipher is (m^e) in $\mathbb{Z}/n\mathbb{Z}$

To decrypt, m^e , raise it to the d^{th} power

A	B	M
p, q	m	n
k		e
d		$c = m^e$
in		

Attack 5: Computing $e^{-1} \bmod k$