2024 WINTER ALGEBRA SEMINAR: MOTIVIC COHOMOLOGY

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Overview

This winter, we will discuss intersection theory and motivic cohomology, with a focus towards geometric applications. The seminar will meet in MS 5138 on Wednesday at 3PM.

Outline of Talks

1. Introduction and Motivation. In this talk, we introduce the main topics of the seminar, providing a brief overview of some of the topics

2. Intersection Theory. To motivate what is to come, we introduce intersection theory in algebraic geometry, namely the basic theory of Chow groups. A sample reference is 3264 and All That.

3. Computations. Continuing on from the previous week, this talk uses Chow groups to answer explicit questions from algebraic geometry, such as counting lines on hypersurfaces. 3264 and All That has a lot of potential applications to talk about.

4. Basics of Higher Chow Groups and Applications. This talk will introduce higher Chow groups and show us how to use higher Chow groups to answer concrete problems about classical Chow groups themselves. For example, prove theorem 1.18 in 3264 and All That on a basis for Chow groups coming from a cell decomposition. Or, discuss a paper of Eric Larson's computing the integral Chow ring of $\overline{\mathcal{M}}_2$.

5. Chow Groups of Classifying Spaces. Continuing to give applications of Chow groups, this talk will focus on a paper of Totaro determining the Chow groups of classifying spaces arising in algebraic geometry.

6. Categories of Motives. This talk will introduce us to categories of motives via correspondences, and discuss how this can be used to define a natural extension of Chow groups: motivic cohomology. A sample reference is Voevodsky.

7. Motivic Cohomology of Fields. Here, we will discuss the statement of the norm-residue isomorphism theorem, how mod ℓ motivic cohomology relates to étale cohomology. Then we will use the theorem to do some computations of motivic cohomology over finite fields field. One possible source is Gabriel Angelini-Knoll's masters thesis for these computations.

8. Motives and Abelian Varieties. This talk discusses a particular subcategory of the category of motives, which is equivalent to the category of abelian varieties with isogenies inverted. A source that could be useful is the following survey by Scholl.

9. Grothendieck's Standard Conjectures. Finally, we discuss Grothendieck's standard conjectures on algebraic cycles, and how the existence of a motivic t-structure implies them, following Beilenson.

General Resources

On Intersection Theory

- 3264 and All That, by Eisenbud and Harris
- Intersection Theory, by Fulton
- The Algebraic and Geometric Theory of Bilinear Forms, by Elmen-Karpenko-Merkurjev

Motivic Cohomology

- The Algebraic and Geometric Theory of Bilinear Forms, by Elmen-Karpenko-Merkurjev
- Burt Totaro's Thesis
- The Norm Residue Theorem in Motivic Cohomology, by Haesemeyer and Weibel
- Classical Motives, by Scholl
- Lecture Notes on Motivic Cohomology, by Mazza-Voevodsky-Weibel.