Mathematics 33A, Practice Midterm, November 2, 2009.

Calculators, books, or notes of any kind are not allowed on the exam.

There are 12 items on this practice exam altogether, and they all have equal value. Answer as many of them as you can in **50 minutes**. You must show your work in all questions.

The questions are not always arranged in order of difficulty. Look through them when you start so you get an idea of the time you'll need. If you're not sure what to do on an item then move onward and return to it later.

After taking the practice test, grade yourself as follows: For each fully correct answer give yourself 5 points. For a score at the A range you should aim to have fully correct answers for at least 9 questions, within the allotted 50 minutes.

Good luck.

Question 1. Find the coordinates of $\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ with respect to the basis $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$.

Question 2. Find an orthonormal basis for $span\{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}\}$.

Question 3. Find a basis for the plane $x_1 + 2x_2 + 3x_3 = 0$.

Question 4. Let A be a 4×4 matrix with columns $\vec{v}_1, \ldots, \vec{v}_4$. We are told that $\vec{v}_1 + 2\vec{v}_2 + 3\vec{v}_3 + 4\vec{v}_4 = 0$. What are the possible values of $rank(A)$? Explain why.

Question 5. Working in $\mathbb{R}^5$ let W be the subspace of all $\vec{x}$ so that $x_1 + x_2 + x_3 + x_4 + x_5 = 0$ and $2x_1 + x_2 + 2x_3 + x_4 + 2x_5 = 0$. Find the dimension of W.

Question 6. Give an example of a basis $\mathcal{B} = \{\vec{v}_1, \vec{v}_2\}$ of $\mathbb{R}^2$ so that $\left[\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right]_{\mathcal{B}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Question 7. Let $\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and let $\vec{v}_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$. Let L be the line through the origin in the direction of $\vec{v}_1$. Let T be the orthogonal projection to L.

(a) Find $T(\vec{v}_1)$ and $T(\vec{v}_2)$. (You can reason geometrically.) Draw $\vec{v}_1$, $\vec{v}_2$, L, $T(\vec{v}_1)$, and $T(\vec{v}_2)$.

(b) Find the matrix of T with respect to the basis $\mathcal{B} = \{\vec{v}_1, \vec{v}_2\}$.

Question 8. Let A be an invertible 2×2 matrix and let $B = \begin{pmatrix} 3 & 2 & 1 \\ 4 & 2 & 2 \end{pmatrix}$. Find the kernel of B. Then find the kernel of AB (and explain how you got it).

Question 9. You are told that the 3×3 matrix $A = (\vec{v}_1 \vec{v}_2 \vec{v}_3)$ is orthogonal, and that $\vec{v}_1$ and $\vec{v}_2$ both lie on the plane $x_1 + 2x_2 + 3x_3 = 0$. Find $\vec{v}_3$.

Question 10. Let A be the matrix $\begin{pmatrix} 3 & 0 \\ 4 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) Find a vector $\vec{y}$ in $image(A)$ minimizing the length of $\vec{y} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

(b) Find a vector $\vec{x}$ minimizing the length of $A\vec{x} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.