Mathematics 33A, Second Midterm, November 6, 2009.

Calculators, books, or notes of any kind are not allowed on this exam. Do not use any paper other than that provided. (You may write on the back if you need more space, but indicate this clearly on the front.)

There are 12 items on this exam altogether, and they all have equal value. Answer as many of them as you can. Show your work and explain your reasoning.

The questions are not always arranged in order of difficulty. Look through them when you start so you get an idea of the time you'll need. If you're not sure what to do on an item then move onward and return to it later.

Good luck.

Family Name: ______________________

Given names: ______________________

Discussion section: ______________________

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Question 1. Find a basis for the kernel of the matrix $A = \begin{pmatrix} -1 & 2 & -3 & 0 \\ 1 & 0 & 2 & -1 \end{pmatrix}$.

Question 2. Find k so that the matrices $\begin{pmatrix} 1 & 2 & 0 \\ 1 & 0 & -2 \\ 1 & 1 & -1 \end{pmatrix}$ and $\begin{pmatrix} 3 & 1 & 0 \\ 1 & -1 & -4 \\ 2 & 0 & k \end{pmatrix}$ have the same image.

Question 3. Let T from $\mathbb{R}^5$ to $\mathbb{R}^5$ be orthogonal projection to $span\{\begin{pmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \\ 2 \end{pmatrix}\}$. Find the dimension of $kernel(T)$.

Question 4. Let $\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$, and $\vec{v}_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$. Let $W = span\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

(a) Using Gram-Schmidt, find an orthonormal basis $\vec{w}_1, \vec{w}_2, \vec{w}_3$ for W.

(b) Write each $\vec{v}_i$ as a linear combination of $\vec{w}_1, \vec{w}_2, \vec{w}_3$, and find the QR decomposition of $\begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{pmatrix}$.

(c) Find a (non-zero) vector $\vec{x}$ in $W^{\perp}$.

Question 5. In this question, $\vec{v}_1$ and $\vec{v}_2$ are vectors in $\mathbb{R}^2$, and you are told that $\vec{v}_i \cdot \vec{v}_j$ is the entry a_{ij} of the matrix $\begin{pmatrix} 3 & 5 \\ 5 & 7 \end{pmatrix}$. L is the line spanned by $\vec{v}_1$.

(a) Find the $\{\vec{v}_1, \vec{v}_2\}$ coordinates of $proj_L(\vec{v}_2)$.

(b) Find the $\{\vec{v}_1, \vec{v}_2\}$ coordinates of the reflection of $\vec{v}_2$ about L.

(c) Find the $\{\vec{v_1}, \vec{v}_2\}$ matrix of reflection about L.

Question 6 Find the orthogonal projection of $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ to the plane in $\mathbb{R}^3$ spanned by $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix}$.

Question 7. Find the least square solution $\vec{x}^*$ to the system $\begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 1 & 4 \\ 2 & 3 \end{pmatrix} \vec{x} = \begin{pmatrix} 0 \\ 0 \\ 3 \\ 1 \end{pmatrix}$.

Question 8. Find a point on the plane $x_1 + x_2 + x_3 = 5$ which is closest possible to the origin.