

Math 245C Homework 3

1. Let  $f : \mathbf{R}^n \rightarrow \mathbf{R}$  be a nonnegative, Borel measurable function which vanishes at infinity, and let  $f^*$  be the symmetric-decreasing rearrangement of  $f$ .

(a) Prove that  $\{x : f^*(x) > t\} = \{x : f(x) > t\}^*$ .

(b) Show that if  $\Phi : \mathbf{R}^+ \rightarrow \mathbf{R}^+$  is nondecreasing, then  $(\Phi \circ f)^* = \Phi \circ f^*$ .

2. Give an example of a function  $g$ : continuous and goes to zero such that there is no  $f \in L^1(\mathbf{R})$  for which  $\hat{f} = g$ .  
(You may use the fact that  $\lim_{N \rightarrow \infty} \left| \int_0^N \frac{\sin x}{x} dx \right| < \infty$ .)

3. Suppose  $f \in L^1(\mathbf{R}^n) \cap L^\infty(\mathbf{R}^n)$  and  $\hat{f} > 0$ . Prove that  $\hat{f} \in L^1$ .

4. Suppose  $f \in L^1(\mathbf{R}^n)$  and  $g \in L^2(\mathbf{R}^n)$ , and that  $\hat{f} = \hat{g}$ . Prove that  $f = g$ .

5. S-S, Chapter 5, Exercise 1.

6. S-S, Chapter 5, Exercise 2.