Homework 5

Due Wednesday, May 8.

(1) Hatcher, Section 2.1, Exercise 3.

(2) (40 pts) Compute $H_i^{\Delta}(X)$ for all $i \geq 0$, where X is: (a) S^n, (b) $\mathbf{RP}^n$, (c) a Klein bottle, (d) a compact surface of genus $g = 2$, and (e) the real line $\mathbf{R}$. (You may pick your favorite Δ-complex structure on each.)

(3) Let $C_* = \{\dots \xrightarrow{\partial} C_n \xrightarrow{\partial} C_{n-1} \xrightarrow{\partial} \cdots\}$ and $D_* = \{\dots \xrightarrow{\partial} D_n \xrightarrow{\partial} D_{n-1} \xrightarrow{\partial} \cdots\}$ be chain complexes and $f : C_* \to D_*$ be a *chain map*, i.e., $f\partial = \partial f$. Then prove that f induces a homomorphism between the homology groups $H_n(C_*)$ and $H_n(D_*)$ of the two complexes.

(4) Using the same notation as in the previous problem, prove that if $f, g : C_* \to D_*$ are chain homotopic chain maps, then f and g induce the same homomorphism on homology.

(5) Hatcher, Section 2.1, Exercises 7,11,12,13 (see p. 110 for the definition of the *reduced homology groups*).