Homework 7

(1) (10 pts each) Guillemin and Pollack, Chapter 3, Section 2, Exercises 2, 3; Chapter 3, Section 3, Exercises 3, 9, 10 (only use intersection theory, not cohomology), 11, 14, 17; Chapter 2, Section 4, Exercises 6, 14, 15, 19.

(2) (40 pts, extra credit) Let M be a noncompact n-dimensional manifold.
   (a) Let $\{U_\alpha\}$ be an open cover of M. Then prove the existence of a countable partition of unity ϕ_i, $i = 1, 2, \ldots$, where each ϕ_i has compact support contained in some U_α. (Hint: Warner, pp. 8–11.)
   (b) Show that M admits a smooth proper map $f : M \to \mathbb{R}$. (Hint: Guillemin-Pollack, p. 53.)
   (c) Show that M admits a smooth proper Morse function $f : M \to \mathbb{R}$.
   (d) Show that M admits an embedding into $\mathbb{R}^{2n+1}$. (Hint: Guillemin-Pollack, p. 54.)