Light review (optional)

1) Let the plane \((P)\) in \(\mathbb{R}^3\) be defined by \(x + y + 2z = 4\) and \(A\) be the point \((1, 1, 4)\). Find the point on \((P)\) that is the closest to \(A\) (called the projection of \(A\) onto \((P)\)).

2) Define \(\text{det} : \mathbb{R}^4 \rightarrow \mathbb{R}\) and \(\text{tr} : \mathbb{R}^4 \rightarrow \mathbb{R}\):

\[
(a, b, c, d) \mapsto ad - bc \quad \quad (a, b, c, d) \mapsto a + d
\]

Let \(I = (1, 0, 0, 1)\) and \(A\) be in \(\mathbb{R}^4\). Prove

\[
\frac{d}{d\varepsilon} \bigg|_{\varepsilon = 0} \text{det} (I + \varepsilon A) = \text{tr}(A)
\]

Remark: This implies \(\text{det}'(I) = \text{tr}\) (linearization of \(\text{det}\) at \(I\)), which is a simple version of [Jacobi's formula](https://en.wikipedia.org/wiki/Jacobi%27s_formula) in linear algebra.

3) Calculate \(\int_0^1 \sqrt{1 - x^2} \, dx\) and \(\int_0^1 x \sqrt{1 - x^2} \, dx\)

4) Calculate
\[
\int_0^1 \int_0^{\sqrt{4 - y^2}} \frac{2y}{x} \, dx \, dy
\]

5) Let \(E = \{(x, y) \mid 0 \leq x, 1 \leq x^2 + y^2 \leq 4\}\)

Calculate \(\iint_E y \, dA\) and \(\iint_E 2x \, dA\)

6) Calculate
\[
\iint_F \frac{\sin(y)}{y} \, dA
\]

where \(F\) is