7th discussion

Let \( A \) bounded set \( \subseteq \mathbb{R}^n. \)

\[ \forall \varepsilon > 0, \text{ there is an inside simple set } F_\varepsilon \]

and outside simple set \( g, \text{ s.t. } \mathcal{V}(F_\varepsilon) - \mathcal{V}(g) \leq \varepsilon. \]

\( C \) is negligible.

\( \Rightarrow \) \( C \) is negligible.

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An open, bounded set is Jordan measurable.

Theorem \( \text{ of } \mathcal{J} \).

Fad Center set

Surface area of closed rectangles

A "simple" set which is negligible must have

Jordan measure zero (compactness).

Some for Jordan measurable sets, (194)

"Almost simple.

Set: \( g : C \rightarrow \mathbb{R} \) is \( C^1 \), \( C \subseteq \mathbb{R}^n \) open

Then \( f : \mathbb{R}^n \rightarrow \mathbb{R}^m \) is negligible \( \Rightarrow \)

i.e. bounded \( \Rightarrow \) \( f^* \) is not surjective.

Proof: Will set \( n \leq 2 \) for sake of clarity

\[ C = \{ C_1, C_2 \} \cup C_2. \]

\[ C_1 \subseteq C. \]

\[ C_2 = \{ y \in C : f^*(y) \subseteq C \} \subseteq \mathbb{R}^n \] f.\( f^*(y) \subseteq C \)

\[ f(y) = f(x) + f^*(y), (y-x) = \frac{f^d(y)}{2} (y-x)^2 + \varepsilon (y-x) (y-x)^2. \]

Equidistant \( \text{ from } f^*(y) \) is negligible \( \Rightarrow \)

Consider \( C_2 \) \( \lambda \) \( \text{ or } \lambda \text{-cube length } R \).

Taylor: \( \varepsilon \), \( \epsilon \in 0 \), \( 0 \leq |f(y) - f(x)| \leq \frac{1}{R} \frac{R^2}{2} |\varepsilon (y-x)|. \)

Done if \( f^*(C_2 \cap C_2) \) is negligible.

Partition \( g \) into \( n \) smaller cubes.

Only care for \( n \)-cube \( C_2 \), \( \text{ where } \lambda \text{-cube } \).

\( \varepsilon \text{ - are } \text{ smaller cubes, \( C_2 \).} \)

\( f \) were into \( \mathcal{R} \) such that \( \varepsilon \text{ - imply } (f^*(\text{small cubes})) \leq \left( \frac{R^2}{n} \left| \frac{R^2}{2} \right| \right. \)

\( \Rightarrow \)
Then \( \text{vol } y(c, M_y) \subset \mathbb{C} \text{ vol } (y \text{ manifold near } c) \)

\[ z \in \left( \frac{\mathbb{D}^n}{\partial \mathbb{D}^n} \right) \cap \partial \mathbb{D}^n \]

\( z \) is arbitrary. Let \( n \to 0 \). Done.

Proof: \( \int (x, y, s) \) is negligible, \( \forall \) as same for \( L \times 0 \), \( L \times y \neq 0 \)

Done: \( \int (x, y, s) \) is negligible

Then \( \int (x, y, s) \) is negligible

In implicit function theorem

as \( \partial \) is injective in \( y \)-direction

Pick a \( c \in E \), IFT near \( a \), any \( b \) is nearby \( x \) also

\( \{ y = 0 \} \),

\( \{ z = 0 \} \) maps into horizontal line \((R)\)

Initial of working on \( U \) and \( f \), work

on flattened \( V \) and \( f' \). (Define polar everything)

As \( a \) is critical point of \( f \), \( f(a) \) is critical point of \( f' \).

To apply induction, must reduce \( f \) to \( R \to R \) function in the obvious way

Recall \( \phi(a) = (\phi(x), 0) \)

\[ f'(x) = f'(x, 0) \]

Define \( \tilde{f}(x) = f(x, 0) \). So \( \tilde{f} : c \to R \text{ (right setting) } \)

Then \( \tilde{f}(x) \) is also such point of \( f_0 \)

\[ \tilde{f}'(x) = 0 \]

Some: \( V \) is \( \tilde{f} \) near \( y \) like \( a \), \( \tilde{f}'(x) \) is also critical point of \( f_0 \).

Then apply \( \tilde{f} \) to \( f_0 \).

\[ \int f(x, y) = e^{x^2 y^2} \text{ on } [-1, 1]^2 \]

\[ \int f(x, y) \text{ d}x \text{ dy } \]

\[ \int f(x, y) \text{ d}x \text{ d}y \]