If you get a “Redo” mark, that means you didn’t get full score for some double star questions, and will have to resubmit them (not the whole homework set).
You can ask me in office hours about how you should rewrite your argument.

**Homework 1**

1.12.6
Please note that \( f \circ g_1 = f \circ g_2 \) is NOT the same as \( f \circ g_1(c) = f \circ g_2(c) \) for some \( c \). It means \( f \circ g_1(c) = f \circ g_2(c) \) for **ALL** \( c \).

1.12.7
Same as above, but for \( h_1 \circ f = h_2 \circ f \).

1.12.8
Be careful of circular logic. You are trying to prove the set can be enumerated by natural numbers, so you can’t just enumerate it as a subsequence.

2.17.1
You can prove this directly by combinatorics. Or indirectly by induction. If you use induction, you MUST make sure to mention that a subset **either** contains the new element, **or** doesn’t contain it. It might sound pedantic, but this **either/or** thing makes sure our list is exhaustive.

2.17.3
You have to state, and prove the explicit Fibonacci formula, either by induction or characteristic equations and matrices.

2.17.5
Do not forget to show the connection between strong and weak induction.

2.17.6
You can actually prove this directly by combinatorics, but I guess people just love writing really long formulas.
For the algebraic properties, I can forgive people who forget about associativity and others, but you must at least mention commutativity of addition and multiplication, as well as distributivity.

**Bonus \( \mathbb{Z}^+ \times \mathbb{Z}^+ \) question**

Just draw the zigzag, or use the injective map \((x, y) \mapsto 2^x 3^y\). Someone mentioned the map \( 2^x + 3^y \). The injectivity of this map is not covered by the Fundamental Theorem of Arithmetic. It’s not even true.

Fun challenge (not hard): can you find a counterexample to show that
\[(x, y) \mapsto 2^x + 3^y\]

is not injective?

**Homework 2**

From homework 2 onwards, only the double star problems, and \(n\) random non-starred problems will be graded, where \(n \in \{0, 1, 2\}\) depending on the week. If I see people start skipping problems, I’ll grade them. The grades for each homework set will be converted to percentages for averaging in the end.

3**

This is Euclid’s lemma for gaussian integers, which is used to prove the uniqueness of prime factorization, so you can’t use prime factorization here. Also, don’t waste too much time proving trivial things like separating \(N(x) = 1\) into 4 cases \(x = 1, -1, i, -i\) ....

5**

The set of lines is not a familiar object. The answer is much more obvious, and more important to algebra.

7**

The whole point here is to notice that a divisor of \(mn\) must be a product of a divisor of \(m\) and a divisor of \(n\). You **MUST** prove this, e.g. by prime factorization or going through the whole Euclid’s lemma thing again if you’re a masochist.

**Homework 3**

4**

Don’t forget about invertibility. And also recall that \(H \leq G\) iff \(\forall x, y \in H : xy^{-1} \in H\).

5**

For transitivity, just say that \(a(bHb^{-1})a^{-1} = (ab)H(ab)^{-1}\). Don’t write pages for this problem.

7

Use combinatorics. Don’t just list all the rotations that you can find online (as you can’t prove these are all the symmetries).

8 choices for \(\sigma(0)\), then 3 for \(\sigma((1,0,0))\), then 2 for \(\sigma((0,1,0))\), then 1 for \(\sigma((0,0,1))\). At this point \(\sigma\) is completely determined. So \(|G| = 48\).
Homework 4

By this point, there is a common theme where people will over-explain things that don’t need explaining while leaving out the details that need to be proved, or write the wrong logic quantifiers (for all vs. there exists etc.), or employ over-complicated ways to prove very simple things. I often don’t want to make you redo everything, and will overlook small problems, but there’s a limit for everything. Notice that when I say your proof is too long, you do not lose any points (unless you’re actually wrong somewhere), but do try to think about what you can do to make the proof more concise and readable, highlighting the most important steps.

3**

Most did fine here. A cool trick is to define the inverse map, bypassing the proof of injectivity and surjectivity.

10.16.2**

People who use induction, or well-ordering, or centralizers and Cauchy’s theorem for this problem can make the proof hard to read. You can prove this directly with Lagrange’s theorem.

10.16.6**

Don’t forget the infinite case, or forget to say the cosets are disjoint.

Let $G = \bigcup_{i \in I} g_i H$ and $H = \bigcup_{j \in J} h_j K$, where $\bigcup$ stands for disjoint union (or $\bigvee$ in Elman’s book). Then

$$G = \bigcup_{i \in I} g_i H = \bigcup_{i \in I} g_i \left( \bigcup_{j \in J} h_j K \right) = \bigcup_{i \in I, j \in J} g_i h_j K$$

as $x \mapsto g_i x$ is bijection on $G$ (think about why I need to say this). Done.