

Speaker: Ragnar Sigurdsson, University of Iceland.

Title: A generalization of the Bernstein-Walsh-Siciak theorem on uniform approximation on compact sets of functions by polynomials.

Abstract: I would like to begin my lecture by sharing a few memories from my time of study in Lund 1978-84. When I arrived, Lars Gårding was running a seminar called “Logaritmen för absolutbeloppet” and was later called “Seminar i klassisk analys”.

I gave my first lecture in Swedish in the seminar in 1979 on Bernstein’s theorem from 1912. It states that a continuous function f on $[-1, 1]$ can be approximated uniformly on $[-1, 1]$ by polynomials in such a way that the minimal error of approximation $d_m(f) = \min\{\|f - p\|_{[-1,1]}; p \in \mathcal{P}_m(\mathbb{C})\}$ in uniform norm on $[-1, 1]$ satisfies $\overline{\lim}_{m \rightarrow \infty} \sqrt[m]{d_m(f)} \leq 1/R$ for some $R > 1$ if and only if f has holomorphic extension to the domain bounded by the ellipse with foci -1 and 1 and semi-major axis R . J.L. Walsh generalized Bernstein’s theorem to approximation on a general simply connected compact K and Siciak generalized it to higher dimensions.

In the lecture I will present a generalization of the Bernstein-Walsh-Siciak theorem to the case when the uniform norm has a weight and the polynomial ring in several variables $\mathcal{P}(\mathbb{C}^n)$ is replaced by the subring $\mathcal{P}^S(\mathbb{C}^n) = \cup_{m \in \mathbb{N}} \mathcal{P}_m^S(\mathbb{C}^n)$, where S is a prescribed compact convex subset of \mathbb{R}_+^n with $0 \in S$ and $\mathcal{P}_m^S(\mathbb{C}^n)$ is the space of all polynomials of the form $\sum_{\alpha \in mS} a_\alpha z^\alpha$. This is a report on a joint work with Benedikt Steinar Magnússon, Alfreður Edda Sigurðardóttir, and Bergur Snorrason.