

Errata as of June 26th, 2007
***p*-Adic Automorphic Forms on Shimura Varieties**
 Springer Monographs in Mathematics

Here is a table of misprints in the above book, and “P.3 L.5b” indicates fifth line from the bottom of the page three. The latest version of the correction table can be downloaded from www.math.ucla.edu/~hida. Addenda (and comments) to the text follow the misprint table. Since some people indicated me that the proofs of the irreducibility theorems of the Igusa tower are difficult, I added more comments (see Addenda).

page and line	Read	Should Read
P.6 L.2b	automorphic factor	automorphy factor or factor of automorphy (the words: ”automorphic factor” may not be a correct mathematical usage).
P.40 L.15b	\mathcal{V}_P	\mathcal{V}_P modulo \mathfrak{m}_P^n for each n
P.41 L.16b	in R .	in R if P is unramified in R .
P.41 L.9b	$P = (x - \alpha) \in \mathbf{P}(K)$	$P = (x - \alpha) \in \mathbf{P}(\overline{K})$
P.42 L.3	of the valuation ring.	of the valuation ring over $\mathcal{V}_P \cap K[x]$.
P.47 L.7b	$\mathfrak{K}_P = K[[t]]$	$R_P = K[[t]]$
P.48 L.4	$\text{Res}_P(f dt_P)$	$\text{Res}_P(f dt_p)$
P.48 L.16	$\text{div}(\phi)$	$\text{deg}(\text{div}(\phi))$
P.48 L.9b	$[K(P) : K]$	$d = [K(P) : K]$
P.48 L.8b	for points $P_1, \dots, P_g \dots$ in $\overline{V}(\overline{K})$,	there are points P_1, \dots, P_g in $\overline{V}(\overline{K})$ with
P.48 L.7b	$v_{P_j} _{\mathfrak{K}} = v_P$	$\mathcal{V}_{P_j} \cap \mathfrak{K} = \mathcal{V}_P$
P.48 L.3b	(trace of P_j)	(trace of P)
P.50 L.11b	\mathfrak{K}	K
P.59 L.4	by coordinates	by all coordinates
P.59 (div)	$\sum_i c_i = 0$ for integers c_i	integers c_i with $\sum_i c_i = 0$
P.62 L.4	$(cu + d, \alpha(z))$	$((cz + d)u, \alpha(z))$
P.63 L.13b	$g^2/g^3 - 27g^2$	$g^2/(g^3 - 27g^2)$
P.63 L.1b	$\sum_{n=1}^{\infty} \frac{(\zeta_N^{bn} q^{an} + \zeta_N^{-bn} q^{-an}) n q^n}{1 - q^n}$	$\sum_{n=1}^{\infty} \frac{(\zeta_N^{bn} q^{anN} + \zeta_N^{-bn} q^{-anN}) n q^{nN}}{1 - q^{nN}}$.
P.65 L.8	$\Gamma_S = S \cap SL_2(\mathbb{Q})$	$\Gamma_S = S \cap PSL_2(\mathbb{Q})$
P.68 L.15b	three conditions:	three conditions (cf. [GME] 2.2):
P.69 L.3	$\mathbf{0}^\#$	$\mathbf{0}^*$
P.72 L.9	$\langle P, \phi^*(Q) \rangle$;	$\langle P, \phi^*(Q) \rangle$ for $\phi \in \text{End}(E/S)$;
P.75 L.16b	fields with	fields k with
P.76 (G2)	$f((E, \phi_N, \omega)_{/R} \times_R R')$	$f((E, \phi_N, \omega)_{/R} \times_{R, \rho} R')$
P.80 L.5	$\text{Gal}(\mathbb{Z}/N\mathbb{Z})/\{\pm 1\}$	$GL_2(\mathbb{Z}/N\mathbb{Z})/\{\pm 1\}$
P.81 L.17b	the cusps)	the cusps; see [GME] 2.5)
P.84 L.6b	(see Theorem 2.40)	(see Theorem 2.43)
P.87 L.5	on $f(E, \phi_p, \phi_N)$	of (E, ϕ_p, ϕ_N)
P.88 L.4b	induced	obtained
P.89 L.5,9	inducing	which restrict to
P.89 L.11b		insert: for a field of fractions K of W
P.89 (F')	independently	uniformly

page and line	Read	Should Read
		Hilbert AVRVM Moduli
P.98 4.1	Hilbert–Blumenthal Moduli	(It would have been better to call this section as “Hilbert AVRVM Moduli” for some reason).
P.103 L.3-4	$(a \otimes \zeta, b \otimes m) \mapsto \mathbf{e}(\mathrm{Tr}_{F/\mathbb{Q}}(ab))\zeta^m$	$\langle (a \otimes \zeta, b \otimes m), (a' \otimes \zeta', b' \otimes m') \rangle_N$ $\mapsto \mathbf{e}(\mathrm{Tr}_{F/\mathbb{Q}}(ab' - a'b))\zeta^{m'-m}$
P.134 L.10	$e_\lambda : V \wedge_F V \cong F$	$e_\lambda : H_1(A, \mathbb{Q}) \wedge_F H_1(F, \mathbb{Q}) \cong F$
P.139 L.17	$Sh_K(G, X)_{/\mathbb{Q}}$	$Sh_K^{(\Sigma)}(G, X)_{/\mathbb{Q}}$
P.149 L.14b	$G(\mathbb{Q}_\Sigma) = B(\mathbb{Z}_\Sigma)G(\mathbb{Z}_\Sigma)$	$G(\mathbb{Q}_\Sigma) = B(\mathbb{Q}_\Sigma)G(\mathbb{Z}_\Sigma)$
P.161 L.15b, 12b, 20b	$G(\mathbb{A}^{(p\infty)})$	$G_1(\mathbb{A}^{(p\infty)})$
P.185 L.3	$S = \mathfrak{M}(\mathfrak{c}, \Gamma_0(\mathfrak{N}))$ (for a lift E of the Hasse invariant)	$T_{\infty, \infty}$
P.185 L.4	S_1	$T_{1, \infty}$
P.191 L.17b	$\sqrt{\chi}$	$\sqrt{\chi}$ with $\sqrt{\chi} \equiv 1 \pmod{\mathfrak{m}_W}$
P.196 (4.76)	$\mathbf{e}_\mathbb{A}$	It may be better to add the definition of $\mathbf{e}_\mathbb{A}$ as the additive character of $F \setminus F_\mathbb{A}$ with $\mathbf{e}_\mathbb{A}(x_\infty) = \mathbf{e}_F(x_\infty)$.
P.196 L.5b	$\bigsqcup_{u \in \mathfrak{o}_q/\mathfrak{q}}$	$\bigsqcup_{u \in \mathfrak{o}_q^{-1}/\mathfrak{q}\mathfrak{o}_q^{-1}}$
P.196 (4.77)	An assumption is missing.	Add “if $\kappa_1 = 0$ ” in (4.77) as an assumption of the formula.
P.197 L.3	$G_\kappa(\mathfrak{N}, \varepsilon; \mathbb{C})$	$S_\kappa(\mathfrak{N}, \varepsilon; \mathbb{C})$
P.200 L.10	$-S_0^D(\mathfrak{N})_{\alpha_1} S_0^D(\mathfrak{N})$	$S_0^D(\mathfrak{N})_{\alpha_1} S_0^D(\mathfrak{N})$
P.201 (SB1)	$u \in S_0^D(\mathfrak{N})C_i$	$u \in S_0^D(\mathfrak{N})C_1^D$
P.205 (4.95)	H^0	H^1
P.206 Thm 4.36	$\varepsilon_+(z^{(p)})$	$\varepsilon_+(z^{(\infty)})$
P.206 L.8	geometric fiber	geometric point
P.216 L.16b	locally	étale locally
P.277 (A3)		It is better to regard $\bar{\eta}$ as a section of the sheaf quotient of $S' \mapsto \mathrm{Isom}(V_{\mathbb{A}(\infty)/S'}, V(A/S'))$ by K over S , where S' runs over the small étale site over S .
P.292 L.16	$\eta : V_{\mathbb{A}(\infty)} \cong V(A)$	
P.299 L.14b	T_∞°/S°	$\mathcal{T}_\infty^\circ/S^\circ$
P.301 L.4	$\mathrm{Hom}_S(A_\xi[p^\infty]^{et}, \mathbb{Q}_p/\mathbb{Z}_p)$ $\cong \mathrm{Hom}_S(A_\xi[p^\infty]^{et}, \mathbb{Q}_p/\mathbb{Z}_p)$	$\mathrm{Hom}_S(A[p^\infty]^{et}, \mathbb{Q}_p/\mathbb{Z}_p)$ $\cong \mathrm{Hom}_S(A_\xi[p^\infty]^{et}, \mathbb{Q}_p/\mathbb{Z}_p)$
P.301 L.5	$\mathrm{Hom}_S(A[p^\infty]^\circ, \mu_{p^\infty})$ $\cong \mathrm{Hom}_S(A_\xi[p^\infty]^{et}, \mu_{p^\infty})$	$\mathrm{Hom}_S(A[p^\infty]^\circ, \mu_{p^\infty})$ $\cong \mathrm{Hom}_S(A_\xi[p^\infty]^\circ, \mu_{p^\infty})$
P.301 L.13	[DAV] V.4.3	[DAV] VII.4.3
P.301 L.8b	$\alpha_m = \begin{pmatrix} 1_n & 0 \\ 0 & p^m 1_n \end{pmatrix}$	$\alpha_m = \begin{pmatrix} 1_n & 0 \\ 0 & p^m 1_n \end{pmatrix}$ and $K_m = K_{\alpha_m}$
P.317 L.13	$Sh_K^{(p)}$	$Sh_{K/\mathbb{F}}^{(p)}$
P.318 L.8b	$(V_A, \langle \cdot, \cdot \rangle_\lambda)$	$(V_A \otimes_{\mathbb{Q}} k, \langle \cdot, \cdot \rangle_\lambda)$
P.319 L.12	ZZU	ZGU
P.320 L.3	$O_C = \{x \in C \mid xL = L\}$	$O_C = \{x \in C \mid xL \subset L\}$
P.320 L.16	$h \in C$	$h' \in C$

page and line	Read	Should Read
P.348 Theorem 8.8	In the heuristic argument in the proof on the lift of a given CM abelian variety over a finite field, add the assumption that A_0 is ordinary	
P.350 (8.16)	Remove “ (R) ” from each term	
P.359 L.13b	$\mathcal{O}_{\widehat{S}_\ell} \otimes_{\mathbb{Z}_p} \mathbf{A}[p^\ell]^{et}$	$\mathcal{O}_{\widehat{S}_\ell} \otimes_{\mathbb{Z}_p} \varepsilon \mathbf{A}[p^\ell]^{et}$
P.364 L.13b	see Theorem 6.27	see Theorem 6.28
P.367 L.16b	localization of	localization at
P.372 L.19	complete intersection	smooth complete intersection
P.377 [SFT]	Bredon	Bredon

Addenda and Comments

P.82 L.5–11: Strictly speaking, φ of $(E, \varphi : \mu_{p^\alpha} \hookrightarrow E[p^\alpha], \phi_N) \in \mathcal{E}_\alpha^{ord}(R)$ induces by Cartier duality that $(\varphi^*)^{-1} : \mathbb{Z}/p^\alpha\mathbb{Z} \cong E[p^\alpha]^{et}$ and hence a point $(\varphi^*)^{-1}(1) \in (E[p^\alpha]^{et} - E[p^{\alpha-1}]^{et})(R)$, which gives rise to the corresponding point $\iota(\varphi) \in (\mathbf{E}[p^\alpha]^{et} - \mathbf{E}[p^{\alpha-1}]^{et})(R)$. Thus we get $\mathcal{E}_\alpha^{ord}(R) \cong (\mathbf{E}[p^\alpha]^{et} - \mathbf{E}[p^{\alpha-1}]^{et})(R)$. As for $(E, P, \phi_N) \in \mathcal{E}'_\alpha^{ord}(R)$, we can associate to P the étale subgroup $\langle P \rangle \subset E[p^\alpha](R)$ isomorphic to $\mathbb{Z}/p^\alpha\mathbb{Z}$. Then by Cartier duality, we get a canonical isomorphism $\varphi' : \mu_{p^\alpha} \cong E[p^\alpha]/\langle P \rangle$, which gives rise to $(E/\langle P \rangle, \varphi', \phi_N) \in \mathcal{E}^{ord}(R)$. In this way, we get $\mathcal{E}'^{ord} \cong \mathcal{E}^{ord}$.

P.85 L.14: For $m > n$, we have $V_{m,\infty} \otimes_W W_n \cong V_{n,\infty}$ by the flatness of $T_{m,\alpha}$ over S_m . In this way, we get the projective system $\{V_{m,\infty}\}_m$. Tensoring $V_{m,\infty}$ with the inclusion $W_n = p^{-n}W/W \hookrightarrow p^{-m}W/W \cong W_m$, we get the inclusion $V_{n,\infty} \hookrightarrow V_{m,\infty}$. Out of these inclusions, we get the injective limit \mathcal{V} .

Corollary 3.5: The point here is that q is the parameter well defined over W of the geometrically irreducible (formal) scheme $T = T_{\infty,\infty} = \mathrm{Spf}(V)$. In other words, by the existence of Tate curve described in [AME] Chapter 8 (or [GME] 2.5), we can compactify T adding the cusp ∞ into a smooth formal scheme \overline{T} , and $T - \overline{T} \cong \mathrm{Spf}(W[[q]])$. Since $T/R = T \widehat{\otimes}_W R$, this gives rise the q -expansion principle.

P.87 (3.3): The first equality is the difficult one. Indeed $H^0(S_{0/W}, \underline{\omega}^k)$ is of infinite rank (because S_0 is affine of relative dimension 1 over W), while, for compactification $X_1(N)$, $H^0(X_1(N)_{0/W}, \underline{\omega}^k)$ is free of finite rank over W (since $X_1(N)$ is projective over W). The second equality is just by the definition of G_k^{ord} .

Irreducibility theorems of the Igusa tower: In the proof given for the Hilbert modular irreducibility theorem: Theorem 4.21, to show the action of the derived group $G_1(\mathbb{A}^{(p^\infty)})$ preserves the valuation v , we only need the commutative diagram in the middle of page 161, since the set $\pi_0(T_n^\circ)$ of the connected components of T_n° for finite $n > 0$ is a finite abelian group isomorphic to a quotient of $(O/p^n O)^\times$ (where the identity of $(O/p^n O)^\times$ is sent to C_∞). This point is mentioned in the paragraph just by two last lines, but they might be a bit difficult (for some people) to understand. Indeed, the action of $G_1(\mathbb{A}^{(p^\infty)})$ on $\pi_0(T_n^\circ)$ gives a homomorphism $\varphi : G_1(\mathbb{A}^{(p^\infty)}) \rightarrow \pi_0(T_n^\circ)$, but $G_1(\mathbb{A}^{(p^\infty)}) = SL_2(F_{\mathbb{A}}^{(p^\infty)})$ does not have nontrivial abelian quotient; so, $\varphi = 1$, which means that C_∞ and the valuation v is kept by the action of $G_1(\mathbb{A}^{(p^\infty)})$.

As indicated at the bottom paragraph of page 161, one can also use the p -adic density of prime-to- p level cusp forms in [H02] and [H03a] in the space of p -adic modular forms of p -level p^∞ in order to show stability of C_∞ and v under $G_1(\mathbb{A}^{(p^\infty)})$.

We can add a similar explanation to the proof of the irreducibility theorem in the Siegel modular case (Theorem 6.27). By the same commutative diagram in page

299, the action of $G_1(\mathbb{A}^{(p^\infty)})$ on the finite set $\pi_0(\mathcal{T}_\alpha^\circ)$ for an integer $\alpha > 0$ gives rise to a group homomorphism $\varphi : G_1(\mathbb{A}^{(p^\infty)}) \rightarrow \text{Aut}(\pi_0(\mathcal{T}_\alpha^\circ))$. The permutation group $\text{Aut}(\pi_0(\mathcal{T}_\alpha^\circ))$ on the finite set $\pi_0(\mathcal{T}_\alpha^\circ)$ is a finite group. Note that the group $G_1(\mathbb{A}^{(p^\infty)}) = Sp_{2n}(\mathbb{A}^{(p^\infty)})$ does not have non-trivial finite quotient group, because $Sp_{2n}(k)$ for a field k of characteristic 0 is generated by unipotent subgroups (which are divisible; see, for example, [H03b] Propositions 3.1 and 3.4). Thus φ is trivial, and hence C_∞° is kept by the action of $G_1(\mathbb{A}^{(p^\infty)})$.

The same proof applies to the (modulo p) Igusa tower over the Shimura variety for $G = \text{Res}_{F/\mathbb{Q}} GSp(2n)$ and $G = \text{Res}_{F/\mathbb{Q}} GU(n, n)$ for any totally real field F (unramified at p) as indicated in [H03a] Corollary 10.2 ([H03a] is now published in *Astérisque* 298 (2005), and [H03b] also in *Documenta Math.* 11 (2006)).

As in the proof of Theorem 4.21, we also referred in the proof of Theorem 6.27 to the density of prime-to- p level cusp forms of parallel weight $\det(\underline{\omega})^{\otimes k}$ at line 6 in page 300 to show the stability of C_∞° under $G_1(\mathbb{A}^{(p^\infty)})$, but (this is a slip of the pen, and) actually for this comment, all weights are necessary (parallel weight cusp forms only may not form a dense subspace in the space). On the other hand, the density theorem is proven in the book and in [H02] Corollary 3.4 for V_{cusp}^N but not for the entire V_{cusp} . We need a similar result for the entire V_{cusp} (whose proof is probably not published yet but of course is doable). Anyway, the proof was already complete (even before reaching this point) by the group theoretic argument (as indicated in the book and also supplemented as above).

A new preprint containing a direct proof of Theorem 8.16 and Corollary 8.17 via an argument similar to the one proving Theorems 4.21 and 6.27 is posted below in the preprint section:

H. Hida, Irreducibility of the Igusa tower, 2006

See also the lecture note of a Luminy talk posted just after the above paper: Irreducibility of the Siegel–Igusa tower.

P.348 Theorem 8.8: In the “heuristic” proof, we need to add the ordinarity assumption on the abelian variety A_0 . Then we have a canonical lift of the CM abelian variety to the base \mathcal{B} . Otherwise, we could have trouble lifting A_0 to a CM abelian variety. See recent paper by Chai–Conrad–Oort: “CM lifting of abelian varieties” posted in Conrad’s web page (<http://www.math.lsa.umich.edu/~bdconrad/>).