Errata and Addenda as of May 1, 2020
Hilbert Modular Forms and Iwasawa Theory
Oxford University Press, 2006

Here is a table of misprints in the above book, and "P.3 L.5b" indicates fifth line from the bottom of the page three. If more misprints are found, this errata table will be updated, and a latest version will be posted in the web page `www.math.ucla.edu/~hida`.

page and line	Read	Should Read
P.viii 1.9	$T \otimes_R \overline{\mathbb{Q}}$	$T \otimes_A \overline{\mathbb{Q}}$
P.3 L.3	p-profinite algebras	p-profinite W-algebras
P.5 L.4	residue field $\mathbb{F}$	residue field $\mathbb{F}$ and fraction field K
P.9 L.5	$P^{e(P)}$	$P^{\ell_P(X)}$
P.12 L.18b	Ind_F^K	Ind_K^F
P.12 L.9b	$\rho : \mathfrak{G}_F \to A$	$\rho : \mathfrak{G}_F \to A^\times$
P.12 L.1b	$\mathfrak{G}_\mathbb{Q}/[\mathfrak{G}_\mathbb{Q}, \mathfrak{G}_\mathbb{Q}]$	$[\mathfrak{G}_\mathbb{Q}, \mathfrak{G}_\mathbb{Q}]$
P.14 L.8		Remove " $\bmod \mathfrak{m}_A v$"
P.17 (uv)	$\tau \cong \overline{\tau}$	$\tau \equiv \overline{\tau}$
P.17 L.18	map. We	map. Extend φ to $\mathbb{F}$ by $\varphi(0) := 0$. We
P.17 (w2)	$X(s,t) - X(t,s)$	$X(s,t),\ X(t,s)$
P.21 L.8	$\mathrm{Hom}(\pi_1(X), A)$	$\mathrm{Hom}(\pi_1(V), A)$
P.21 L.15b	$\pi_1^{alg}(V)/\overline{(\pi_1^{alg}(V_{/\overline{\mathbb{Q}}}), \pi_1^{alg}(V_{/\overline{\mathbb{Q}}})))}$	$\pi_1^{alg}(V_{/\overline{\mathbb{Q}}})/\overline{(\pi_1^{alg}(V_{/\overline{\mathbb{Q}}}), \pi_1^{alg}(V_{/\overline{\mathbb{Q}}}))}$
P.21 L.10b	$\mathrm{Hom}(\pi_1(\mathbb{Z}), \mathbb{Z}_\ell)$	$\mathrm{Hom}(\mathbb{Z}, \mathbb{Z}_\ell)$
P.22 L.4b	$((2\pi i)\mathbb{Q})^{\otimes m} \otimes H_B^m$	$((2\pi i)\mathbb{Q})^{\otimes m} \otimes H_B^n$
P.23 L.18b	$\int_Z \omega$	$\int_V \omega$
P.24 L.9b, L15b	$H^n(V)$ (three places)	$H^n(V)(m)$
P.28 L.10	$\rho : \mathfrak{G}_F \to GL_2(K)$	$\rho : \mathfrak{G}_F^S \to GL_2(K)$
P.29 L.1b	$F_\mathfrak{q}$	$F_\mathfrak{p}$
P.34 L.4b	$\boldsymbol{\rho}_\mathfrak{p}\vert_{D_\mathfrak{p}}$	$\boldsymbol{\rho}_F\vert_{D_\mathfrak{p}}$
P.35 L.17	End_K	End_A
P.39 Below 1.54		Insert: "Suppose that $\mathbb{I}$ is normal."
P.39 L.18	$\mathcal{R}_F$	$\widetilde{\mathcal{R}}_F$
P.42 L.3b	$\mathrm{Gal}(K/\mathbb{Q})$	$\mathrm{Gal}(K/F)$
P.43 L.1,2,8,9,10	$\mathrm{Gal}(K/\mathbb{Q})$	$\mathrm{Gal}(K/F)$
P.44 L.17b	may other	many other
P.51 L.5	the corollary	Proposition 1.67
P.90 L.6	$S_A = S_{\overline{A}} \sqcup S_{A_f}$	$S_A \supset S_{\overline{A}} \sqcup S_{A_f}$
P.99 L.1b	$x^{-(\kappa_1+\kappa_2)+I}$	$x_\infty^{-(\kappa_1+\kappa_2)+I}$
P.101 L.5b	$a \in \mathfrak{a}^*$	$\xi \in \mathfrak{a}^*$
P.101 L.3b	$\mathrm{Tr}_{F/\mathbb{Q}}(\xi O)$	$\mathrm{Tr}_{F/\mathbb{Q}}(\xi \mathfrak{a})$
P.102 (2.3.7)	$a \in \mathfrak{a}_+^*$	$\xi \in \mathfrak{a}_+^*$
P.103 (ex2)	ε_v^-	ε^-
P.103 (ex3)	$Z(\mathbb{A}^{(\infty)})/Z(\mathbb{Q})$	$Z(\mathbb{A}^{(\infty)})$
P.103 (ex3)	$Z(\mathbb{A})S_0((v) \cap \mathfrak{N}) = Z(\mathbb{A})S_1((v) \cap \mathfrak{N}) \to \mathcal{B}^\times$	$Z(\mathbb{A})S_0((v) \cap \mathfrak{N}) \to \mathbb{C}^\times$
P.103 (2.3.13)		Here U is the unipotent radical of the upper triangular Borel subgroup of GL_2.
P.103 (2.3.14)	h	u, t
P.107 L.7	Insert after $\varepsilon = (\varepsilon_+, \varepsilon_1, \varepsilon_2)$	on $Z(\mathbb{A}^{(\infty)})$ and $T(\widehat{\mathbb{Z}})$
P.110 L.13	firms	forms
P.114 L.15	$d(B)$ (two places)	$d(D)$
P.117 L.4b	$\int_\mathcal{U} \pi(u) v du$	$\int_\mathcal{U} \pi(u) w du$
P.118 (2.3.31)	$\lambda(b)g(g)$	$\lambda(b)f(g)$
P.118 L.17	$\phi(bxb^{-1})$	$\phi(bub^{-1})$

page and line	Read	Should Read
P.120 L.3	Insert at the end	if $S \supset \mathcal{B}(O_{\mathfrak{q}})$
P.120 L.13	if $S \supset B(\mathbb{Z}_p)$	if $S \supset \mathcal{B}(O_{\mathfrak{q}})$
P.120 L.21	$\xi = \left(\begin{smallmatrix} \varpi^{a_1} & 0 \\ 0 & \varpi^{a_2} \end{smallmatrix}\right)$	$\xi = \left(\begin{smallmatrix} \varpi^{a_1} & 0 \\ 0 & \varpi^{a_2} \end{smallmatrix}\right)$ with $a_1 \geq a_2$
P.126 Theorem 2.43 (2)	$\varepsilon_+\mathcal{N}^{[\kappa]}$	$\varepsilon_+\mathcal{N}$
P.127 L.1b	$k(Q)$	$k(P)$
P.129 2.45	Replace $\mathbb{T}$ by $\mathbf{T}$ in the proof	
P.129 L14	characteristic P	characteristic p
P.132 L.7	$\big\lvert c \in N\widehat{\mathbb{Z}}$	$\big\lvert a\widehat{\mathbb{Z}} + N\widehat{\mathbb{Z}} = \widehat{\mathbb{Z}},\ c \in N\widehat{\mathbb{Z}}$
P.132 (2.4.6)	$\sum_{0<d\mid(m,n)}$	$\sum_{0<d\mid(m,n),(d,N)=1}$
P.133 L.10b	$\phi \in \mathcal{H}_k(\Gamma_0(N);\mathbb{Z})$	$\phi \in \mathrm{Hom}(\mathcal{H}_k(\Gamma_0(N);\mathbb{Z}),\mathbb{Z})$
P.141 (2.5.2)	$\Theta_{ij}(\phi(a_j),\phi^*(a_i))$	$\Theta_{ij}(\phi(a_j)\otimes\phi^*(a_i))$
P.183 L.8b	$\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$	$\mathrm{Gal}(\overline{\mathbb{Q}}/F)$
P.183 L.8b	$\mathfrak{c}(\varepsilon)p$	$\mathfrak{N}p$
P.185 L.4	the finite flat group scheme $(\mathbb{Z}/p\mathbb{Z})\oplus\mu_p)\otimes_{\mathbb{F}_p}\mathbb{F}$	a finite flat group scheme, and $\overline{\rho}^{ss}$ is given by the twist of $(\mathbb{Z}/p\mathbb{Z})\oplus\mu_p)\otimes_{\mathbb{F}_p}\mathbb{F}$
P.188 L.8b	$=\mathbb{F}[\epsilon]\otimes_{\mathbb{F}}\varprojlim_n \frac{O_{\mathfrak{p}}^{unr,\times}}{(O_{\mathfrak{p}}^{unr,\times})^{p^n}}$ (RHS)	$=\mathbb{F}[\epsilon]\otimes_{\mathbb{F}}\varprojlim_n \frac{O_{\mathfrak{p}}^{\times}}{(O_{\mathfrak{p}}^{\times})^{p^n}}$
P.190 Thm 3.23 2.	*the number of*	*the number r of*
P.201 L.11, 16	$\prod_{\mathfrak{q}\in\Sigma_{\overline{\rho}}\sqcup Q}$	$\prod_{\mathfrak{q}\in\Sigma_{\overline{\rho}}}$
P.202 L.6b	$H^0(I_{\mathfrak{q}},Ad(\overline{\rho}))/(Frob_{\mathfrak{q}}-1)Ad(\overline{\rho})$	$H^0(I_{\mathfrak{q}},Ad(\overline{\rho}))/(Frob_{\mathfrak{q}}-1)H^0(I_{\mathfrak{q}},Ad(\overline{\rho}))$
P.203 L.13	$\dim_{\mathbb{F}}(O_{\mathfrak{p}}/pO_{\mathfrak{p}})-1$ if $\alpha_{\mathfrak{p}}=\overline{\mathcal{N}}_{\mathfrak{p}}$	$\dim_{\mathbb{F}}(O_{\mathfrak{p}}/pO_{\mathfrak{p}})+1$ if $\alpha_{\mathfrak{p}}=\overline{\mathcal{N}}_{\mathfrak{p}}$
P.207 L.14	$\dim_{\mathbb{F}}\mathrm{Sel}^{\perp}_{\emptyset}(Ad(\overline{\rho}))=\dim_{\mathbb{F}}\Phi_Q(\mathbb{F}[\epsilon])$	$\dim_{\mathbb{F}}\mathrm{Sel}^{\perp}_{\emptyset}(Ad(\overline{\rho})(1))\geq\dim_{\mathbb{F}}\Phi_Q(\mathbb{F}[\epsilon])$
P.209 L.20b	σ acts by the identity on X.	σ has eigenvalue 1 on X.
P.214 L.6	$\prod_{\mathfrak{q}\in\Sigma^{\overline{\rho}}}\frac{H^1(F_{\mathfrak{q}},Ad(\rho_0)_m)}{\Phi_{P,\mathfrak{q}}(\mathbb{F}[\epsilon])}$	$\prod_{\mathfrak{q}\in\Sigma^{\overline{\rho}}}\frac{H^1(F_{\mathfrak{q}},Ad(\rho_0)_m)}{\Phi_{P,\mathfrak{q}}(W_m[\epsilon])}$
P.224 L.3	F-version of Kummer's criterion	See Addenda for a proof.
P.248 L.15b	(E1–3)	(E2–3)
P.249 Theorem 3.69, Lemma 3.70	Add the following assumption.	If a prime $\mathfrak{l}\nmid p$ ramifies in F'/F the ramification index of $\mathfrak{l}$ is prime to p.
P.265 L.6b	Insert the follwing after "such factors."	Decompose $Y=\mathcal{Y}\oplus K^{t_0}\oplus K(1)^{t_1}$ as D_p-modules so that $\mathcal{Y}^{D_p}=0=\mathcal{Y}(1)_{D_p}$.
P.264 L.9b	$\prod_{\mathcal{I}/D_p}$	$\prod_{\mathrm{Hom}_{\mathrm{field}}(F,\overline{\mathbb{Q}})/D_p}$
P.266 L.7b	$\mathcal{F}^+H^1(\mathbb{Q}_p,Y)$	$\mathcal{F}^+H^1(\mathbb{Q}_p,\mathcal{Y})\times\mathrm{Hom}(D_p/I_p,K)^{t_0}\times H^1_{fl}(\mathbb{Q}_p,K(1))^{t_1}$ $\subset H^1(\mathbb{Q}_p,Y)$
P.356 Lemma 5.7 (i)	$\Omega_{A/R}\otimes_R C$, $\Omega_{C/A}$, $\Omega_{C/R}$	$\Omega_{A/B}\otimes_A R$, $\Omega_{R/B}$, $\Omega_{R/A}$
P.359 L.2	$\Omega_{\mathcal{R}_j/P_j}\otimes_{\mathcal{R}_j}\mathcal{R}_j/P_j$	$\Omega_{(\mathcal{R}_j/(P_j\cap\Lambda_j)\mathcal{R}_j)/W}\otimes_{\mathcal{R}_j}\mathcal{R}_j/P_j$
P.359 L.3	$\Omega_{\mathcal{R}_j/P_j/\mathcal{R}_j}\otimes_{\mathcal{R}_j}\mathcal{R}_j/P_j$	$\Omega_{(\mathcal{R}_j/(P_j\cap\Lambda_j)\mathcal{R}_j)/W}\otimes_{\mathcal{R}_j}\mathcal{R}_j/P_j$
P.359 L.10	$\lambda_h=W[[y_{\mathfrak{p}}]]_{\mathfrak{p}}$	$\Lambda_h=W[[y_{\mathfrak{p}}]]_{\mathfrak{p}}$
P.361 L.16	Replace R and R_n by R_P and $R_{n,P}$	
P.365 L.14	$(y_1,\dots,f_{d_n})$	$(y_1,\dots,y_{d_n})$
P.367 L.16	$\mathbb{I}$-module	$\mathcal{R}_0$-module
P.372 L.5b	Remove "as $\psi^A_{n/B}$"	
P.377 L.9b	the fixed subfield of Γ^- (resp. Γ^+)	the fixed subfield of Γ^+ (resp. Γ^-)
P.380 L.12	$\widetilde{\varphi}^-(\sigma)=\widetilde{\varphi}(c\sigma c^{-1}\sigma)$	$\widetilde{\varphi}^-(\sigma)=\widetilde{\varphi}(c\sigma c^{-1}\sigma^{-1})$
P.384 L.10b	sign error	place "-1" in front of all the exponents
P.384 L.7b	$-e(\mathfrak{p})\log_p(N_{\mathfrak{p}}(\alpha(\mathfrak{P})_{\mathfrak{P}}^{-e(\mathfrak{p})}))$.	$-e(\mathfrak{p})\log_p(N_{\mathfrak{p}}(\alpha(\mathfrak{P})_{\mathfrak{P}}))$.
Lemma 5.37	$\alpha(\mathfrak{P})^{1-c}_{\mathfrak{P}'}$	$\alpha(\mathfrak{P})^{c-1}_{\mathfrak{P}'}$
P.385 L.3	$\delta_{\mathfrak{p}}([\gamma_{\mathfrak{p}},F_{\mathfrak{p}}])$	$\frac{\delta_{\mathfrak{p}}([\gamma_{\mathfrak{p}},F_{\mathfrak{p}}])}{[F_{\mathfrak{p}}:\mathbb{Q}_p]}$
Theorem 5.38 Corollary 5.39	$\prod_{\mathfrak{p}\mid p}\frac{e(\mathfrak{p})}{h}$	$(-1)^e\prod_{\mathfrak{p}\mid p}\frac{1}{h\cdot f_{\mathfrak{p}}}$ for $f_{\mathfrak{p}}=[O/\mathfrak{p}:\mathbb{F}_p]$, because of the correction at P.385 L.3

Addenda/Errata

- P.26 L.3: Here is some history of the rationality theory of critical L-values (not touched in the book). It is important to have researchers entering into this area know how rationality theory of L-values actually developed, and I have decided to add some explanation (not to misguide new researchers by the short statement starting at line 3 in page 26). The conjectures in [D4] were made only after Shimura had established a couple of years earlier rationality for modular and Hecke L-values. The theory goes back to Euler (in the eighteenth century) for the critical Riemann zeta values and to Siegel (and Klingen) for critical Dedekind zeta values. The modern theory for modular and automorphic L-values was started by Shimura in his early paper [59c] Section 9 (in [CPS] volume I) for the critical values of $L(s,\Delta)$ (Δ is Ramanujan's function of weight 12). In his later papers [75c] [76b] and [77d] (in [CPS] II), he established rationality of Hecke L-values in [75c] and rationality of general elliptic modular critical L-values in [76b] and [77d]. One of his main ideas in these works (and later ones) is the use of certain nonholomorphic differential operators acting on automorphic forms which preserve rationality (but not holomorphy) of automorphic forms and theta functions (up to explicit constants; for example, [75c] and [77c]). If we move an evaluation point (of an automorphic L-value) by integers (within the critical range), out of experience, one might guess that, in many cases (if not all), the move adds (or eliminates) a power of $2\pi i$ to (or from) the transcendental factor (the period) of the starting L-value. The precise move of the exponent of $2\pi i$ in the period was proven by Shimura in many cases (for example, [76b]) using often this property of the differential operator (one can find a motivic interpretation of this move of the exponent of $2\pi i$ for motivic L-values in a later paper [D4] of Deligne). Further in [77c] Remark 3.4 (in [CPS] II), periods (up to algebraic numbers) of rational differentials on abelian varieties with real multiplication were determined in terms of the values of a certain rational meromorphic Hilbert modular form over the field of real multiplication. This result provides the equivalence between the rationality result of the Hecke L-values proven in [75c] and the rationality of the Hecke L-values with respect to an appropriate CM period (which is also discussed later in [D4]). A preprint of [77b] in the proceedings of an international conference in Kyoto held in 1976 was circulated among the participants of the conference (including the author of this note). The paper [77b] contains in particular as Theorem 4 the rationality theorem in [76b] and [77d]. After these works, Deligne made his conjecture on the rationality of motivic L-values with respect to his motivic period in a conference at Corvallis (which was held in July-August 1977), and his paper [D4] was later published in 1979 (though, appeared strange in the eyes of the author of this note, Deligne does not quote in [D4] Shimura's earlier works except for [75c]). In [D4], Deligne checked his conjecture conforming well to the known results at the time. After these works, Shimura went on and extended his rationality results (for example, his CM period relation in [79a], his factorization of CM and non CM periods in terms of periods of quaternionic automorphic forms in [83a] and [88]...) even to non-motivic L-values (for example, in [81a] and [88] in [CPS] III, values of L-functions associated to *half-integral* weight modular forms are treated) and to the values of explicitely given automorphic forms and Dirichlet series of new type (for example, [81b,c]). Later from late 1980s, other researchers joined in the rank and started studying rationality of L-values and automorphic forms, and many such rationality results (motivic or non-motivic) so far known have been proven guided by the automorphic methods Shimura invented. Indeed, the proof of the anticyclotomic main conjecture in [H05d] relies on the results on the new type of Dirichlet series in [81b,c].
- P.195 Lemma 3.24 and Theorem 3.25 (This error was pointed out to the author by Olivier Fouquet). Lemma 3.24 and its proof are both wrong, for example, $\mathbb{A}^\times/\mathbb{Q}^\times\mathbb{R}_+^\times \cong \widehat{\mathbb{Z}}^\times/\{\pm 1\}$ has a lot of 2-torsion (in the proof, it is claimed to have no 2-torsion). This lemma is used in the proof of Theorem 3.25 in the middle of page 197. The corrected statement of Theorem 3.25 is

  Theorem 3.25 *Let p be an odd prime, and suppose* (h1–4) *and* ($\mathrm{ai}_{F[\mu_p]}$). *Then a Taylor–Wiles system $\{R_Q, M_Q\}_{Q\in\mathcal{Q}}$ for the universal deformation ring R_Q of Φ_Q exists for an infinite set $\mathcal{Q}$ of finite subsets $Q \subset \Sigma_F$ such that*

  (1) *$\mathcal{Q}$ is made up of finite subsets Q of primes $\mathfrak{q}$ outside $p\mathfrak{c}(\varepsilon)$ satisfying* (tw1) *and* (reg)*;*
  (2) *M_Q is the direct factor of the W-dual space*

  $$\mathrm{Hom}_W(\mathcal{S}^{\Sigma_0\text{-}ord}(S(Q),\varepsilon;W),W)$$

for $S(Q) = \widehat{\Gamma}_0(\mathfrak{N}) \cap \widehat{\Gamma}_1^1(Q)$ *under the Hecke operator action (as specified in the proof) and satisfies the module compatibility condition* (5) *in* Theorem 3.23*;*

(3) *the primes* $\mathfrak{q} \in Q$ *have residual degree* 1*, and the primes* $\mathfrak{q}$ *in* $\bigcup_{Q \in \mathcal{Q}} Q$ *with* $N(\mathfrak{q}) \equiv 1 \mod p^n$ *for any given* $n > 0$ *have positive density.*

The proof of Theorem 3.25 is sound if one removes the paragraph starting at the top of page 197 to line 25 of page 197. As originally stated in Fujiwara's work correctly, replacing the starting phrase of the paragraph beginning at line 12 from bottom of page 197: "Thus assuming (ni)" by "Thus assuming ($\text{ai}_{F[\mu_p]}$)", the proof works well. Lemma 3.24 and the misstated original Theorem 3.25 is never used in the rest of the book, and the version of Theorem 3.25 used in the book is the one corrected as above.

- P.224 L.3: (KCF:F-version of Kummer's criterion) used in the text can be stated as:

> *Let* ϕ *be a totally even Hecke character of* F *of conductor 1 and* p *be an odd prime unramified in* $F/\mathbb{Q}$*. If one has a Hecke eigen cusp form* f *of level 1 whose Hecke eigenvalue for* $T(\mathfrak{l})$ *is congruent to* $1+\phi(\mathfrak{l})N_{F/\mathbb{Q}}(\mathfrak{l})$ *modulo* $\mathfrak{p}$ *for a prime ideal* $\mathfrak{p}|p$ *for all primes* $\mathfrak{l}$ *of* F *outside* p*, then* $p|L(-1,\phi)$*.*

Proof. We relate the L-value $L(-1,\phi)$ to class number of a certain cyclotomic extension. Since ϕ modulo p only matters; so, we may assume that ϕ has order prime to p. Take a character χ of order prime to p such that $\chi^{-1}\omega^{-1} = \phi$ for the Teichmüller character ω modulo p (so, χ is totally odd). Let $F(\chi)$ for the CM field with $\mathrm{Gal}(F(\chi)/F) = \mathrm{Im}(\chi)$. Write $F_+(\chi)$ for the maximal real field of $F(\chi)$. Then $L(-1,\phi) \equiv L(0,\chi^{-1}) \mod p$ (by the existence of Deligne–Ribet p-adic L-function of $\chi^{-1}\omega = \phi\omega^2$).

Remark (not a necessary ingredient of the proof): The product $H = \prod_{j:odd} L(0,\chi^j)$ (j running over odd positive integers up to the order of χ) is basically the relative class number of $F(\chi)/F_+(\chi)$ (at least if $p > 3$ is unramified in F, the p-part of the class number coincides with the p-part of the product). Let $H' = \prod_{j:odd} L(-j,\phi^j)$. Then $H' \equiv H \mod p$ by the same reason as above.

Suppose one has a Hecke eigen cusp form f as above of level 1. Then the Galois representation $\overline{\rho}$ of f modulo $\mathfrak{p}$ is upper triangular. Then by Ribet's argument in [Ri1], we may assume that $\overline{\rho}(\sigma) \equiv \begin{pmatrix} 1 & * \\ 0 & \chi^{-1} \end{pmatrix} \mod \mathfrak{p}$ and that $\overline{\rho}$ is non-semisimple. The splitting field L of $\overline{\rho}$ is a p-abelian extension of $F(\chi)$ unramified everywhere such that $\mathrm{Gal}(F(\chi)/F)$ acts on $\mathrm{Gal}(L/F(\chi))$ by χ. From this, what we find is that $p|H$ or equivalently $p|H'$.

Here is how to show $p|L(-1,\phi)$ (equivalently $p|L(0,\chi^{-1})$) from the result of [Wi1]. Let $F_\infty(\chi)/F(\chi)$ be the cyclotomic $\mathbb{Z}_p$-extension. Note $F \cap \mathbb{Q}_\infty = \mathbb{Q}$ by the unramifiedness of p in $F/\mathbb{Q}$. Write $L(\chi)/F_\infty(\chi)$ be the maximal p-abelian extension unramified everywhere. Let X^χ be the χ-eigenspace of $\mathrm{Gal}(L(\chi)/F_\infty(\chi))$, and write $g(T)$ for the characteristic power series of X^χ as in [Wi1]. Here $T = u-1$ in the Iwasawa algebra Λ for a fixed generator u of $1+p\mathbb{Z}_p$ (identified with $\mathrm{Gal}(F_\infty(\chi)/F(\chi))$. Theorem 1.2 and 1.4 in [Wi1] combined tells us that $g(T)$ is equal to the Deligne-Ribet p-adic L-function $L_p(s,\chi^{-1}\omega)$ (or more precisely, the Iwasawa power series $G_{\chi^{-1}\omega}(u(1+T)^{-1}-1)$ for $G(T)$ giving $L_p(s,\chi^{-1}\omega)$ as modified in [Wi1] page 494).

(KCF) is valid by the following result due to Iwasawa (see Washington's book [ICF] Proposition 13.28):

$$\mathrm{hdim}_\Lambda(X^\chi) = 1$$

(that is, X^χ does not have pseudo-null module non-null). Since ϕ is unramified at p, χ ramifies at p; therefore $L_p(0,\chi^{-1}\omega) \neq 0$ ($L_p(s,\chi^{-1}\omega)$ does not have exceptional zero at $s=0$). Then, by a standard argument in Iwasawa theory and the theory of Fitting ideals combined, the order $|X^\chi/TX^\chi|$ is then equal to $|g(0)|_p^{-1}$ which is equal to $|L_p(0,\chi^{-1}\omega)|_p^{-1} = |L(0,\chi^{-1})|_p^{-1}$. The group X^χ/TX^χ is the χ-eigenspace of the p-primary part of the class group of $F(\chi)$. Thus we have a surjective homomorphism $X^\chi/TX^\chi \twoheadrightarrow \mathrm{Gal}(L/F(\chi))$, which is nontrivial by the Ribet construction. Recall that $L(-1,\phi) \equiv L(0,\chi^{-1}) \mod p$; so, $p|L(-1,\phi)$. □

- P.244 L.25: $\mathrm{Tr}(\rho(\sigma)) \in A_L$, because

$$\mathrm{Tr}(\rho(\sigma)) = \rho(\sigma) + (\det(\rho(\sigma))\rho(\sigma^{-1}) = \rho(\sigma) + \varepsilon_+\mathcal{N}(\sigma)\rho(\sigma^{-1}) \in A \cap \mathrm{End}(L) = A_L.$$

- On the correction (in P.249) for Theorem 3.69 and Lemma 3.70: The added assumption is necessary to guarantee that the extended deformation ρ of ρ' (which exists by [MFG] Lemma 5.32) satisfies the condition (Q6). Indeed, under this condition, the local deformation $\rho|_{I_{\mathfrak{l}}}$ is determined by $\overline{\rho}|_{I_{\mathfrak{l}}}$ and ρ'. Later applications concern only quadratic F'/F or cyclotomic p-extension F'/F unramified outside p; so, this does not affect the arguments using Theorem 3.69 later.
- On the correction at P.266 L.7b: The space $\mathcal{F}^+H^1(\mathbb{Q}_p,\mathcal{Y})\times \mathrm{Hom}(D_p/I_p,K))^{t_0}\times H^1_{fl}(\mathbb{Q}_p,K(1))^{t_1}$ can be rewritten as the image of $H^1_{fl}(\mathbb{Q}_p,\mathcal{F}^+Y)\times \mathrm{Hom}(D_p/I_p,Y^{D_p})$ since $\mathcal{F}^+Y\times Y^{D_p}\subset Y$. The latter expression is independent of the choice of the decomposition $Y=\mathcal{Y}\oplus K^{t_0}\oplus K(1)^{t_1}$ inserted at P.265 L6b. By this expression, $\dim(\mathcal{F}^{00}H^1_p(M)/\overline{U}_p(M))=t+t_0+t_1$ claimed at P.268 L.6 is clear because $\dim\mathcal{Y}=2t$. Obviously, $\overline{U}_p(\mathrm{Ind}_F^{\mathbb{Q}}V)=U_p(\mathrm{Ind}_F^{\mathbb{Q}}V)$ if $t_1=t=0$ which is the case when $V=Ad(\rho_f)$ for a nearly p-ordinary Hilbert cusp form f which is not Steinberg at any prime factors of p. If f is Steinberg at a prime factor of p, $t>0$ but $t_1=0$ always, and still we have $\overline{U}_p(\mathrm{Ind}_F^{\mathbb{Q}}V)=U_p(\mathrm{Ind}_F^{\mathbb{Q}}V)$ (see [H2] Lemma 1.6 and [H3] Lemma 1.4) so, this is harmless for the application in the book.
- Lemma 3.83: For any ring A and the trivial G-module A, if we write $\iota: H^1(H,A)\cong H^1(G,\mathrm{Ind}_H^G A)$ for the isomorphism of Shapiro's lemma, we have $\iota=\mathrm{Res}^{-1}$. Indeed, $H^q(\overline{G},A[\overline{G}])=0$ for all $q>0$ ($\overline{G}=G/H$), and from $H^0(\overline{G},H^1(H,A[\overline{G}]))=H^0(\overline{G},H^1(H,A)\otimes_A A[\overline{G}])=H^1(H,A)$, the inflation-restriction sequence shows that this lemma is valid for any A not necessarily a field $A=K$ of characteristic 0. Thus if $\sigma\in G$ has order h in $\overline{G}$, for $\phi\in H^1(H,K)=\mathrm{Hom}(H,K)$, $\iota(\phi)(\sigma)=h^{-1}\phi(\sigma^h)$. Since $[p,\mathbb{Q}_p]^{d_{\mathfrak{p}}}=[p,F_{\mathfrak{p}}]|_{\mathbb{Q}_p^{ab}}$, for $\phi\in H^1(F_{\mathfrak{p}},K)=\mathrm{Hom}(\mathrm{Gal}(\overline{F}_{\mathfrak{p}}/F_{\mathfrak{p}}),K)$, applying the above fact to $G=\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$ and $H=\mathrm{Gal}(\overline{\mathbb{Q}}_p/F_{\mathfrak{p}})$, we have $\iota(\phi)([p,\mathbb{Q}_p])=d_{\mathfrak{p}}^{-1}\phi([p,F_{\mathfrak{p}}])$ and $\iota(\phi)([u,\mathbb{Q}_p])=d_{\mathfrak{p}}^{-1}\phi([u,F_{\mathfrak{p}}])$ (for $d_{\mathfrak{p}}=[F_{\mathfrak{p}}:\mathbb{Q}_p]$) as explained in page 273, because
$$d_{\mathfrak{p}}\cdot\iota(\phi)([u,\mathbb{Q}_p])=\iota(\phi)([u^{d_{\mathfrak{p}}},\mathbb{Q}_p])=\mathrm{Res}(\iota(\phi))([u,F_{\mathfrak{p}}])=\phi([u,F_{\mathfrak{p}}]).$$
Therefore $\frac{\iota(\phi)([u,\mathbb{Q}_p])}{\log_p(u)}=d_{\mathfrak{p}}^{-1}\frac{\phi([u,F_{\mathfrak{p}}])}{\log_p(u)}$.
- In Theorem 3.93 (or its proof), a full detailed argument relating $q_{\mathfrak{p}}$ and $N_{F_{\mathfrak{p}}/\mathbb{Q}_p}(Q_{\mathfrak{p}})$ for the Tate period $Q_{\mathfrak{p}}$ at $\mathfrak{p}$ of the elliptic curve E is not given. It can be found in [H1] below. Our formula computes the $\mathcal{L}$-invariant of $\mathrm{Ind}_F^{\mathbb{Q}}Ad(V)$. Though $L(s,Ad(V))=L(s,\mathrm{Ind}_F^{\mathbb{Q}}Ad(V))$, the modification (vanishing) Euler p-factor computed over $\mathbb{Q}$ following [G] (6) and the corresponding Euler $\mathfrak{p}$-factor over F are different if $O/\mathfrak{p}\neq\mathbb{F}_p$. Indeed, the vanishing factor over $\mathbb{Q}_p$ is of degree 1 for each $\mathfrak{p}|p$ and is given by $\prod_{\mathfrak{p}|p}(1-a_{\mathfrak{p}})$ with $a_{\mathfrak{p}}=1!$. However over F at $\mathfrak{p}$, it is of degree $f_{\mathfrak{p}}:=[O/\mathfrak{p}:\mathbb{F}_p]$ and is given by $(1-a_{\mathfrak{p}}^{f_{\mathfrak{p}}})=(1-a_{\mathfrak{p}})\prod_{1\neq\zeta\in\mu_{f_{\mathfrak{p}}}}(1-\zeta a_{\mathfrak{p}})$. Thus we have the following identity of the nonvanishing factor
$$\mathcal{E}_+(\mathrm{Ind}_F^{\mathbb{Q}}Ad(V))=(\prod_{\mathfrak{p}|p}\prod_{1\neq\zeta\in\mu_{f_{\mathfrak{p}}}}(1-\zeta))\mathcal{E}_+(Ad(V)_{/F})=(\prod_{\mathfrak{p}|p}f_{\mathfrak{p}})\mathcal{E}_+(Ad(V)_{/F})$$
under the notation in [H] Conjecture 0.1, because $f_{\mathfrak{p}}=\prod_{1\neq\zeta\in\mu_{f_{\mathfrak{p}}}}(1-\zeta)$. Thus the expression of the $\mathcal{L}$-invariant $\mathcal{L}(Ad(V)_{/F})$ over F is slightly different from $\mathcal{L}(\mathrm{Ind}_F^{\mathbb{Q}}Ad(V))$ over $\mathbb{Q}$, and indeed,
$$\mathcal{L}(Ad(V)_{/F})=(\prod_{\mathfrak{p}|p}f_{\mathfrak{p}})\mathcal{L}(\mathrm{Ind}_F^{\mathbb{Q}}Ad(V)),$$
since we have $\lim_{s\to1}\frac{L_p(s,\rho)}{(s-1)^e}=\mathcal{L}(\rho)\mathcal{E}^+(\rho)\frac{L(1,\rho)}{c^+(\rho(1))}$ conjecturally for $\rho=Ad(V)$ and $\mathrm{Ind}_F^{\mathbb{Q}}Ad(V)$. Then our formula in Theorem 3.93 formulated for $\mathcal{L}(Ad(V)_{/F})$ takes the following shape:
$$\mathcal{L}(Ad(V)_{/F})=\prod_{\mathfrak{p}|p}\frac{\log_p(q_{\mathfrak{p}})}{\mathrm{ord}_{\mathfrak{p}}(Q_{\mathfrak{p}})}\quad(\Leftrightarrow\mathcal{L}(\mathrm{Ind}_F^{\mathbb{Q}}Ad(V))=\prod_{\mathfrak{p}|p}\frac{\log_p(q_{\mathfrak{p}})}{\mathrm{ord}_p(q_{\mathfrak{p}})}),$$
where $\frac{1}{f_{\mathfrak{p}}}\mathrm{ord}_p(q_{\mathfrak{p}})=\mathrm{ord}_{\mathfrak{p}}(Q_{\mathfrak{p}})$. C.-P. Mok (see [M]) computed the analytic $\mathcal{L}$-invariant for some Tate curves E over F following the method of Greenberg–Stevens and got the formula using $\mathrm{ord}_{\mathfrak{p}}(Q_{\mathfrak{p}})$ as above under some assumptions. His formula gives the value $\mathcal{L}(V_{/F})$ as the product of $\frac{\log_p(q_{\mathfrak{p}})}{\mathrm{ord}_{\mathfrak{p}}(Q_{\mathfrak{p}})}$ over p-adic places $\mathfrak{p}$ where E has split multiplicative reduction under some assumptions on $E_{/F}$ and F.

- P.385 L.3: The variable $x_{\mathfrak{p}}$ appearing in this formula is different from the one in Theorem 3.73. If we write the variable $x_{\mathfrak{p}}$ in page 385 as $X_{\mathfrak{p}}$ (and keep writing the variable $x_{\mathfrak{p}}$ in Theorem 3.73, we have the relation $(1+x_{\mathfrak{p}}) = (1+X_{\mathfrak{p}})^{[F_{\mathfrak{p}}:\mathbb{Q}_p]}$; so, $[F_{\mathfrak{p}} : \mathbb{Q}_p]\frac{\partial}{\partial x_{\mathfrak{p}}} = \frac{\partial}{\partial X_{\mathfrak{p}}}$. Thus after the correction as indicated above, the formula is equivalent to the one given in Theorem 3.73. This correction causes the modification of the formulas in Theorem 5.38 and Corollary 5.39.
- Theorem 5.38 and Corollary 5.39: As for the sign error part "$(-1)^e$", a detailed explanation can be found in [H] Section 3.3. By the addendum to Theorem 3.93,
$$\det\left(\left(\frac{\log_p(N_{\mathfrak{p}'}(\alpha(\mathfrak{P})^{c-1}_{\mathfrak{P}'}))}{h}\right)_{\mathfrak{P},\mathfrak{P}'\in\Sigma_p^c}\right)$$
gives the $\mathcal{L}$-invariant $\mathcal{L}(Ad(\mathrm{Ind}_M^F\,\widetilde{\varphi}_P)_{/F}) = \mathcal{L}(\alpha_{/F})$ (for $\alpha = \alpha_{M/F}$) over F.

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