Errata and Addenda as of March 1, 2021
Modular Forms and Galois Cohomology
Cambridge studies in advanced mathematics **69**, 2000

Here is a table of misprints in the above book, and "P.3 L.5b" indicates fifth line from the bottom of the page three. Addenda to the text follow the misprint table.

page and line	Read	Should Read
P.9 L.12b	$\varphi^*: \mathbb{A}^\times/\mathbb{Q}^\times$	$\varphi^*: \mathbb{A}^\times/\mathbb{Q}^\times \to \mathbb{C}^\times$
P.10 L.8b	$\mathbb{R}^\times$	$\mathbb{R}^\times_+$
P.11 L.10b	$\widehat{O} = O \otimes_{\mathbb{Z}} \widehat{Z}$	$\widehat{O} = O \otimes_{\mathbb{Z}} \widehat{\mathbb{Z}}$
P.13 L.11	φ of F	φ of F'
P.13 $(*)$	$\prod_i (1-\alpha_i X)^{-1}$ $= \det(1_n - \rho(\phi)X)^{-1}$	$X\frac{d}{dX}\log(\det(1_d - \rho(\phi)X)^{-1}) + d$
P.14 L.3	$\varphi(\phi_i^{m/f_i}) = 0$	$\widehat{\varphi}(\phi_i^{m/f_i}) = 0$
P.14 L.6, 8	$\sum_{\zeta\in\mu_{f_i}} \zeta\widehat{\varphi}$	$\sum_{\zeta\in\mu_{f_i}} \zeta^m\widehat{\varphi}$
P.14 L.10	$\det(1_n - \rho(\phi)X)$	$\det(1_d - \rho(\phi)X)$
P.19 L.17	the final chapter (5)	the final chapter
P.23 L.6b	$M \supset M_1 \supset \cdots \supset M_n = \{0\}$	$M \supsetneq M_1 \supsetneq \cdots \supsetneq M_n = \{0\}$
P.33 L.3b	$\rho : G \to M_n(E)$	$\rho : G \to GL_n(E)$
P.36 L.6b	$\sigma = \rho$	$\sigma \sim \rho$
P.37 L.6	$\bigoplus_{\rho\in\widehat{G}} e_\rho E[G]$	$\bigoplus_{\rho\in\widehat{G}_E} e_\rho E[G]$
P.40 L.7b	closed subgroup of finite index	closed subgroup
P.40 L.7b	$M \in R_E(H)$	$M \in Rep_E(H)$
P.40 L3b	map:Ind_H^G	functor:Ind_H^G
P.42 L.13b	$R_E(G)$	$R_E(H)$
P.45 L.15b	$\delta : M_n(F) \to M_n(F)$	E-linear $\delta : M_n(E) \to M_n(E)$
P.49 L.10	v_n, $v_{\sigma(n)}$	v_m, $v_{\sigma(m)}$
P.50 L.12		add (2.10) at the end and remove (2.10) three lines below.
P.50 L.2b	$T(r_0r_2r_1)$	$T(r_2r_1r_0)$
P.51 L.11	$\rho : R \to A$	$\rho : R \to M_n(A)$
P.51 L.13	$\overline{\rho} : R \to GL_n(A)$	$\overline{\rho} : R \to M_n(A)$
P.51 (2.11)	$\rho : G \to GL_2(A)$	$\rho : G \to GL_n(A)$
P.52 L.10b	$\mu_{q-1} \cong \mathbb{F}^\times$	$\mu_{q-1}(A) \cong \mathbb{F}^\times$
P.53 L.14b	$\iota_j([g_p]_j) = \rho_j(g_p)$	$\iota_j([g_p]_{i(j)}) = \rho_j(g_p)$
P.53 L.12b	ι	ι_ρ
P.54 L.8	$\varrho : \Gamma \to \mathcal{O}[[T]]$	$\varrho : \Gamma \to \mathcal{O}[[T]]^\times$
P.54 L.13	$\iota(f(T))$	$\iota_\rho(f(T))$
P.54 L.16	the general object $A = \varprojlim_j A_j$	a general object $A = \varprojlim_j A_j$
P.54 L.9b	$G_p \cong \Gamma \cong \mathbb{Z}_p$	$G_p^{ab} \cong \Gamma \cong \mathbb{Z}_p$
P.56 L.8	$\tau \cong \overline{\tau}$	$\tau \equiv \overline{\tau}$
P.57 L.11	Subsection 2.3.2.2	Subsection 2.2.2
P.57 L.8b, 6b, 5b	$\mathfrak{m}$	$\mathfrak{m}_A$
P.59 L.6	tangent	co-tangent
P.59 L.10	$pA = 0$	$\mathfrak{m}_{\mathcal{O}}A = 0$
P.59 L.14	$\mathfrak{m}/\mathfrak{m}_A^2$	$\mathfrak{m}_A/\mathfrak{m}_A^2$
P.60 L.7b	$V(ad(\overline{\rho}))$	$ad(\overline{\rho})$
P.60 L.5b	$\mathfrak{m}$	$\mathfrak{m}_R$
P.60 L.3b	$\phi(r) = \phi_0(a) + \phi_\varepsilon(r)\varepsilon$	$\phi(r) = \phi_0(r) + \phi_\varepsilon(r)\varepsilon$
P.61 L.2	$\overline{a} = a \mod \mathfrak{m}_A$	$\overline{a} = a \mod \mathfrak{m}_R$

page and line	Read	Should Read
P.61 L.14b	$u' : G \to M_n(\mathbb{F})$	$u'_\rho : G \to M_n(\mathbb{F})$
P.61 L.17	mutiplicativity	multiplicativity
P.61 L.6b	$\overline{\rho}(g) + u'_\rho(g) =$ $(1+x\varepsilon)(\overline{\rho}(g)+u'_{\rho'}(g))(1-x\varepsilon)$	$\overline{\rho}(g)+u'_\rho(g)\varepsilon =$ $(1+x\varepsilon)(\overline{\rho}(g)+u'_{\rho'}(g)\varepsilon)(1-x\varepsilon)$
P.62 L.3b	$\dim_{\mathbb{F}} H^1(G, ad(\overline{\rho})) < \infty$	$\dim_{\mathbb{F}} H^1_{ct}(G, ad(\overline{\rho})) < \infty$
P.65 L.1	$\chi(a)$	$\chi(d)$
P.67 L.3b	semi-simple	semi-simple, simply-connected
P.68 L.9	dense in $\mathbb{A}$	dense in $\mathbb{A}^{(\infty)}$
P.75 L.12	$\chi_N^{-1}(\gamma^{(\infty)})f((\gamma u)_\infty)$	$\chi_N^{-1}(\gamma^{(\infty)})f(\gamma \cdot u_\infty)$
P.76 L.16	(A1)for	(A1) for
P.89 L.4b	$Proj(\overline{\mathcal{A}})[\frac{1}{\Delta}]$ affine	$Proj(\overline{\mathcal{A}})[\frac{1}{\Delta}]$ is the affine
P.94 L.18b	given in (1.42)	given in (3.42)
P.94 L12b, 4b	$k \geq 2$	$k > 2$
P.96 L.11	$h(S(\Gamma_0(N),\chi;A))$	$h(S_k(\Gamma_0(N),\chi;A))$
P.97 L.5b	$\mathrm{Hom}_{\mathbb{Z}}(\mathcal{S}(\mathbb{Z})A,\mathbb{Z})$	$\mathrm{Hom}_{\mathbb{Z}}(\mathcal{S}(\mathbb{Z}),\mathbb{Z})$
P.100 L.7b	χ'	$\chi\chi'$
P.107 L.1	$X^2 - t(p)X + p^{k-1}\langle p\rangle$	$X^2 - t(p)X + \langle p\rangle$
P.108 L.19	GL_2	GL_n
P.109 L.2	$\mathbb{Z}_p(\lambda')$	$\mathcal{O} = \mathbb{Z}_p(\lambda')$
P.109 L.14	$\rho_{\lambda'}$	$\rho = \rho_{\lambda'}$
P.110 L.2	diagonal	upper-triangular
P.110 L.7	$\lambda(T(q)(-\mathbb{Z}_p^\times(\lambda'))$	$\lambda(T(q))(\in \mathbb{Z}_p(\lambda')^\times)$
P.114 L.3b		in the second formula replace Tr by det
P.115 L.7	$\mathfrak{G}_{pN}$	$\mathfrak{G} = \mathfrak{G}_{pN}$
P.116 L.6b	h is	h be
P.117 L.2	p	p
P.121 L.18	P	p
P.128 L.9b	Δ_q	Δ_Q
P.136 L.10b	Chebotarev density, theorem using	Chebotarev density theorem and using
P.153 L.2b	$PSL_2(\mathbb{F}) \cong A_4$	$PSL_2(\mathbb{F}_3) \cong A_4$
P.165 L.12b	$(E \times_M M') \times'_M M''$	$(E \times_M M') \times_{M'} M''$
P.167 L.1b	$\tau : S' \to I'$	$\tau : S \to I'$
P.168 L.1	$\phi_2 - \phi'_2 = \pi'_* \circ \tau$	$(\phi_2 - \phi'_2)_* = \pi'_* \circ \tau_*$
P.175 L.1,2	δ_j	δ_{j-1}
P.186 L.5	$\widehat{C}_{p+1}$	$\widehat{C}_{\lvert p+1 \rvert}$
P.186 L.11b	$0 \to \mathbb{Z} \xrightarrow{\varepsilon} C_0 \xrightarrow{\partial_0} C_1$	$0 \leftarrow \mathbb{Z} \xleftarrow{\varepsilon} C_0 \xleftarrow{\partial_0} C_1$
P.186 L.9b	$\widetilde{C}_{-2} \xrightarrow{\delta_{-2}} \widetilde{C}_{-1} \xrightarrow{\varepsilon} \mathbb{Z} \to 0$	$\widetilde{C}_{-2} \xleftarrow{\delta_{-2}} \widetilde{C}_{-1} \xleftarrow{\varepsilon} \mathbb{Z} \leftarrow 0$
P.189 L.13b	The above fact (4.21) and (4.13)	The above facts (4.21) and (4.13)
P.191 L.7	$\prod_{m \in M - \{0\}}$	$\prod_{m \in M^U - \{0\}}$
P.207 L.11	Theorem 4.19	Theorem 4.22
P.209 L.6	$\varprojlim_{U \in \mathcal{U}, \widehat{M}^U = \widehat{M}} H^r(G/U, M \otimes_{\mathbb{Z}} C)$	$\varinjlim_{U \in \mathcal{U}, \widehat{M}^U = \widehat{M}} H^r(G/U, \widehat{M} \otimes_{\mathbb{Z}} C)$
P.210 L.8b	$\mathrm{Ext}_{\mathcal{C}}(\mathbb{Z}/m\mathbb{Z}, C) = 0$	$\mathrm{Ext}^3_{\mathcal{C}}(\mathbb{Z}/m\mathbb{Z}, C) = 0$
P.211 L.16	$H^q_{ct}(U,X)$	$H^q_{ct}(U,X)^*$
P.215 (cf2)	$\begin{array}{ccc} H^2(U,C) & \longrightarrow & H^2(V,C) \\ \wr\downarrow & & \wr\downarrow \\ \mathbb{Q}/\mathbb{Z} & \xrightarrow{x \mapsto nx} & \mathbb{Q}/\mathbb{Z}. \end{array}$	$\begin{array}{ccc} H^2(U,C)[l^\infty] & \longrightarrow & H^2(V,C)[l^\infty] \\ \wr\downarrow & & \wr\downarrow \\ \mathbb{Q}_l/\mathbb{Z}_l & \xrightarrow{x \mapsto nx} & \mathbb{Q}_l/\mathbb{Z}_l \end{array}$ for all $l \in S$.

page and line	Read	Should Read
P.217 L.3	$H^1(\mathfrak{H}, \overline{\mathbb{Q}}^\times) = 0.$	$H^1_{ct}(\mathfrak{H}, \overline{\mathbb{Q}}^\times) = 0.$
P.218 L.5	$rec_S : C_S \to \mathfrak{G}^{ab}_S$	$rec_S : C^{\mathfrak{G}}_S \to \mathfrak{G}^{ab}_S$
Cor. 4.49	See Addenda below	
P.219 L.1	$C^{\mathfrak{G}}_S/mC^{\mathfrak{G}}_S$	$C^{\mathfrak{G}}_S/\ell^m C^{\mathfrak{G}}_S$
P.219 L.3	$\mathfrak{G}^{ab}/m\mathfrak{G}^{ab}$	$\mathfrak{G}^{ab}/\ell^m\mathfrak{G}^{ab}$
P.219 L.6	$C^{\mathfrak{G}}_S/mC^{\mathfrak{G}}_S \to \mathfrak{G}^{ab}/m\mathfrak{G}^{ab}$	$C^{\mathfrak{G}}_S[m] \to \mathfrak{G}^{ab}[m]$ (see also Addenda below)
P.230 middle	see addenda below	
P.231 L.4	$(U/\mu) \otimes_{\mathbb{Z}} W$	$(U/\mu) \otimes_{\mathbb{Z}_p} W$
P.241 Example 5.1		$\mathfrak{H} = \mathrm{Gal}(F^{(p,\infty)}/F)$
P.253 L.9	φ_∞	φ
P.260 L.16b	a A-free	an A-free
P.282–285	$\Omega_1(\pm, \lambda^\circ; A)$	$\Omega(\pm, \lambda^\circ; A)$ (defined in page 274 L.1)
P.285 L.10	$\int_{\mathbb{A}^\times} f(x)\lvert x\rvert^s_{\mathbb{A}} dx$	$\int_{\mathbb{A}^\times} f\left(\left(\begin{smallmatrix} x & 0 \\ 0 & 1\end{smallmatrix}\right)\right)\lvert x\rvert^s_{\mathbb{A}} dx$
P.287 L.1	regular sequence f.	regular sequence f (see [CRT] Section 16).
P.303 L.5b	(no_H)	(NO)
P.304 L.6	(Rg_H)	(Rg_D) $(\forall D \in S_H)$
P.305 L.2	$g^{-1}_{D'} P_D(B) g_{D'}$	$g^{-1}_{D'} B_D(B) g_{D'}$
P.324 L.8	$\begin{array}{ccc} U_\infty \otimes_{\mathbb{Z}_p} \mathbb{I} & \longrightarrow \\ \downarrow & \\ J/J^2 \otimes_{\Lambda_\infty} \mathbb{I} & \longrightarrow \end{array}$	$\begin{array}{ccc} U_\infty \otimes_{\mathbb{Z}_p} \mathbb{I} & \longrightarrow \\ \iota\downarrow & \\ J/J^2 \otimes_{\Lambda_\infty} \mathbb{I} & \longrightarrow \end{array}$
P.340 L.7	(Sh) 209	(III) 220

Corrections and Addenda

- P.29 L.17b: an ordered set X means that if $A, B \in X$, we find C in X such that $A \subset C$ and $B \subset C$.
- P.106 Proof of Lemma 3.25: We should have remarked that $h' = h$ by the Chebotarev density theorem applied to the Galois representations $\rho_{\lambda'}$ (in Theorem 3.26) for λ' factoring through h.
- P.212 L.9b: Between "the duality." and "In this case", add the following explanation:
  In particular, by $\alpha^2(\mu_m)$, we have
  $$\mu_m(K)^* = H^0_{ct}(G, \mu_m)^* \cong \mathrm{Ext}^2_{\mathcal{C}}(\mu_m, C) \cong H^2_{ct}(G, \mathrm{Hom}(\mu_m, C)) = H^2(G, \mathbb{Z}/m\mathbb{Z}).$$
  By the long exact sequence attached to $0 \to \mathbb{Z} \xrightarrow{m} \mathbb{Z} \to \mathbb{Z}/m\mathbb{Z} \to 0$, the cohomology group $H^2_{ct}(G, \mathbb{Z}/m\mathbb{Z})^*$ surjects onto $(H^2_{ct}(G, \mathbb{Z}) \otimes \mathbb{Z}/m\mathbb{Z})^* \cong G^{ab}[m] \cong \mu_m(K)$ by the reciprocity map, and hence $H^2_{ct}(G, \mathbb{Z}/m\mathbb{Z}) = G^{ab}[m]$.
- P.213 L.3b: Add the following explanation to the end of the proof of Theorem 4.43:
  By Proposition 4.30 and the divisibility of C,
  $$H^r_{ct}(G, M) \cong \mathrm{Ext}^r_{\mathcal{C}}(M^*(1), C) = 0$$
  for $M^*(1) = \mathrm{Hom}(M, C)$.
- P.218 Corollary 4.49 (2) should read:
  Suppose that M is a finite module. Let F be a finite totally imaginary Galois extension of K inside K^S such that $\mathrm{Gal}(K^S/F)$ *acts trivially on M and $\mu_{|M|}$.*
- P.219 L.7: Replace "the corollary." by:
  the corollary, because $H^2_{ct}(\mathrm{Gal}(K^S/F), \mathbb{Z}/m\mathbb{Z})^* = H^2_{ct}(\mathrm{Gal}(K^S/F), \mu_m)^* = \mathfrak{G}^{ab}[m]$ is the surjective image of $C^{\mathfrak{G}}_S[m]$ by the theory of Brauer groups [CFT] Chapter 14 (and $F \supset \mu_m$).

The above correction does not affect the sequel, since Corollary 4.49 is quoted only at P.220 L.14 and P.223 L.8 and the relevant statement is (1) of Corollary 4.49.

Here is how to prove the last point using the theory of Brauer groups: Let $\ell \in S$ and write $X_\ell = X[\ell^\infty]$ for a module X. Write also $K_S^{S\times} = \varinjlim_{L\subset K^S} L_S^\times$ and $E_S = \varinjlim_{L\subset K^S} E_{L,S}$. We may assume that $m = \ell^k$ for $0 < k \in \mathbb{Z}$. Then $1 \to E_S \to K_S^{S\times} \to C_S \to 1$ is exact, and hence we have two injections:

$$H^2_{ct}(\mathfrak{H}, E_S)_\ell \hookrightarrow H^2_{ct}(\mathfrak{H}, K_S^{S\times})_\ell \hookrightarrow \prod_{v\in\Sigma} H^2_{ct}(\mathfrak{H}_v, \overline{F}_v)_\ell,$$

where Σ is the set of places of F over places in S, and $\mathfrak{H}_v = \mathrm{Gal}(\overline{F}_v/F_v)$ for an algebraic closure $\overline{F}_v$ of F_v. Thus by using the local duality, $\mathrm{Hom}(\mu_m, K_S^{S\times}) \to H^2_{ct}(\mathfrak{H}, \mu_m)^*$ is surjective. Since $H^2_{ct}(\mathfrak{H}, C_S)_\ell \cong \mathbb{Q}_\ell/\mathbb{Z}_\ell$ (cf2-3), it is not difficult to see that the image of $\mathrm{Hom}_{\mathcal{C}_\mathfrak{H}}(\mu_m, C_S)$ in $\mathrm{Hom}_{\mathcal{C}_\mathfrak{H}}(\mu_m, K_S^{S\times})$ actually covers $H^2_{ct}(\mathfrak{H}, \mu_m)^*$. Thus if F contains all m–th roots of unity, we can replace μ_m by $\mathbb{Z}/m\mathbb{Z}$. □

- P.230 middle: By local Tate duality, $\chi(G, M) = \chi(G, M^*(1))$; so, here, we actually compute

$$\chi(G, M^*(1)) = -\dim_{\mathbb{F}}(\{(F^\times/(F^\times)^p) \otimes_{\mathbb{F}} \kappa\}[\rho]) = \chi(G, M).$$

- P.237 L.15: Here is some history of the rationality theory of critical L-values (not touched much in the book). It is important to have researchers entering into this area know how rationality theory of L-values actually developed, and I have decided to add some explanation (not to misguide new researchers by the short statement starting at line 15 in page 237). The conjectures in [D2] were made only after Shimura had established a couple of years earlier rationality for modular and Hecke L-values. In this note, the reference [CPS] indicates Shimura's collected papers published by Springer in 2002 (there are four volumes quoted as I, II,...). The theory goes back to Euler (in the eighteenth century) for the critical Riemann zeta values and to Siegel (and Klingen) for critical Dedekind zeta values. The modern theory for modular and automorphic L-values was started by Shimura in his early paper [59c] Section 9 (in [CPS] volume I) for the critical values of $L(s, \Delta)$ (Δ is Ramanujan's function of weight 12). In his later papers [75c] [76b] and [77d] (in [CPS] II), he established rationality of Hecke L-values in [75c] and rationality of general elliptic modular critical L-values in [76b] and [77d]. One of his main ideas in these works (and later ones) is the use of certain nonholomorphic differential operators acting on automorphic forms which preserve rationality (but not holomorphy) of automorphic forms and theta functions (up to explicit constants; for example, [75c] and [77c]). If we move an evaluation point (of an automorphic L-value) by integers (within the critical range), out of experience, one might guess that, in many cases (if not all), the move adds (or eliminates) a power of $2\pi i$ to (or from) the transcendental factor (the period) of the starting L-value. The precise move of the exponent of $2\pi i$ in the period was proven by Shimura in many cases (for example, [76b]) using often this property of the differential operator (one can find a motivic interpretation of this move of the exponent of $2\pi i$ for motivic L-values in a later paper [D2] of Deligne). Further in [77c] Remark 3.4 (in [CPS] II), periods (up to algebraic numbers) of rational differentials on abelian varieties with real multiplication were determined in terms of the values of a certain rational meromorphic Hilbert modular form over the field of real multiplication. This result provides the equivalence bewteen the rationality result of the Hecke L-values proven in [75c] and the rationality of the Hecke L-values with respect to an appropriate CM period (which is also discussed later in [D2]). A preprint of [77b] in the proceedings of an international conference in Kyoto held in 1976 was circulated among the participants of the conference (including the author of this note). The paper [77b] contains in particular as Theorem 4 the rationality theorem in [76b] and [77d]. After these works, Deligne made his conjecture on the rationality of motivic L-values with respect to his motivic period in a conference at Corvallis (which was held in July-August 1977), and his paper [D2] was later published in 1979 (though, appeared strange in the eyes of the author of this note, Deligne does not quote in [D2] Shimura's earlier works except for [75c]). In [D2], Deligne checked his conjecture conforming well to the known results at the time. After these works, Shimura went on and extended his rationality results (for example, his CM period relation in [79a], his factorization of CM and non CM periods in terms of periods of quaternionic automorphic forms in [83a] and [88]...) even to non-motivic L-values (for example, in [81a] and [88] in [CPS] III, values of L-functions associated to *half-integral* weight modular forms are treated) and to the values of explicitely given automorphic forms and Dirichlet series of new type (for example, [81b,c]). Later from late 1980s, other researchers joined in the rank and started studying rationality of L-values and automorphic forms, and many such rationality results (motivic or non-motivic) so far known have been proven guided by the automorphic methods Shimura invented.