Math 121

Topics for Midterm II--- Monday, June 4, 2007

General topological spaces (Chapter II of text): definition (set with a specified subset of the set of all subsets of the set, etc.). Subspaces (subset of a topological space "inherits" a topology). Metric space topological ideas extended to general spaces: closed sets, compact sets, etc.   Examples of spaces that are not metric (finite complement, countable complement).

Basis for a topology. Existence of countable basis implies every cover has a countable subcover. "Smallest" topology containing a given collection of subsets as open(arbitrary unions of finite intersections of the given sets that are specified to be open).   Most important "separation" idea: Hausdorff space. Metric spaces are Hausdorff. Examples of topological spaces that are not in general Hausdorff(the finite complement space, the countable complement space). Compactness for general topological spaces: compact subset of a Hausdorff space is closed. Product spaces(definition).

Fundamental group and covering spaces(Chapter III mostly): Closed paths (at a "base point") and homotopy (definition, how to draw the diagrams for proofs) of closed paths (at a basepoint). Definition of the fundamental group $\pi_1(X, x_0)$. Associativity of the product and other group properties(identity, inverse).   Why the fundamental group is independent of base point if X is path connected(i.e., groups associated to different choices of base point are isomorphic). Free homotopy: fundamental group is 0 is equivalent to all loops are freely homotopic to a constant. The fundamental group of the circle is isomorphic to the integers: argument using "winding numbers"(lifting to the "covering space" R over S^1). General idea of covering space and "universal[simply connected] cover". Lifting of homotopies. (arguments use Lebesgue numbers! Recall proof that Lebesgue numbers exist for coverings of compact metric spaces and proof of this).   Higher homotopy groups and why they are independent of base point (same as for fundamental group essentially) and why they are commutative (abelian).