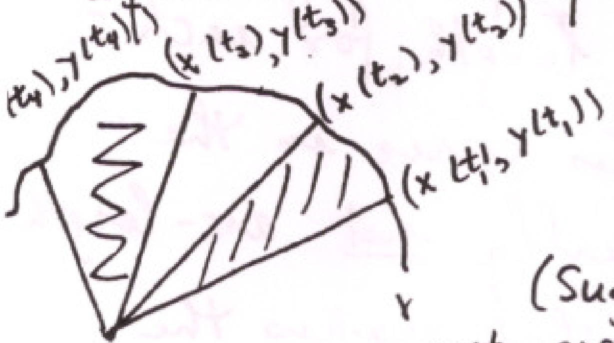
Homework 3: Due Monday, February 4, 2008
FOUR PROBLEMS: SEE BACK SIDE

1. Let $\gamma(t) = (x(t), y(t))$ be a plane curve, not necessarily unit speed, that is never $= (0,0)$. Suppose that "the acceleration is radial", that is, that $(x''(t), y''(t))$ is, for each t, a (vector) multiple of $(x(t), y(t))$, $(x''(t), y''(t)) = \lambda(t)(x(t), y(t))$. Prove that the vector $(x(t), y(t))$ sweeps out equal areas in equal times (see picture)



area \\\ = area ξ

if $t_2 - t_1 = t_4 - t_3$.

(Suggestion: Show that the swept out area $A(t)$ has the property that $\frac{dA}{dt}$ is constant by using area $= \frac{1}{2}\oint(-y\,dx + x\,dy)$ and noting that $\oint -y\,dx + x\,dy$ along a "radius" $= 0$).

2. Continuing problem 1, if $\gamma(t)$ has the form $(r(t), \theta(t))$ in polar coordinates, show that
$$\frac{dA}{dt} = \frac{1}{2} r^2(t) \frac{d\theta}{dt}$$

3(a) Prove that the curve $\gamma(t) = (A\cos t, A\sin t, Bt)$, $A>0$, has curvature $A/(A^2+B^2)$ and torsion $B/(A^2+B^2)$.

(b) Given any two numbers $k_0 > 0$ and τ_0 show that there exist numbers A, B, with $A>0$ such that
$$k_0 = A/(A^2+B^2) \qquad \tau_0 = B/(A^2+B^2)$$

(c) Deduce that every curve with constant curve $k_0 > 0$ and constant torsion τ_0 is (part of) a curve of the form given in part (a).

4. Problem 6.10 (p. 48) in textbook

Curvature part only!

(If $\sigma(s)$ is an arclength-parameter curve then the tangent spherical indicatrix of σ is the curve $T(s)$, which lies on the unit sphere of $\mathbb{R}^3$, since $T(s)$ has unit length, for each s.

Note however that this curve in the unit sphere is usually <u>not</u> arc-length parameter! This is what makes the problem a problem!)