Kirby Diagrams and Framings Exercises

Exercise #1: Let X be a simply-connected 4-manifold with nonempty connected boundary. Show that X is determined by X_2 and the number of 3-handles attached.

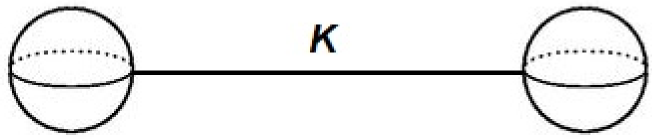

Figure 1: Knot in $S^1 \times S^2$

Exercise #2: (Exercise 4.4.4) For the knot K in $S^1 \times S^2$ shown above, why algebraically does changing a framing of K by two twists not change the isotopy class of the knot's framing? Show geometrically an isotopy of K that changes the framing by two twists? How does this relate to Philippine dancing?

Exercise #3: Let F_1 and F_2 be Seifert surfaces respectively for knots K_1 and K_2 in S^3. Show directly that $F_1 \cdot K_2 = F_2 \cdot K_1$. Conclude that the Seifert surface definition of linking number is well-defined.

Exercise #4: Fill in the details of the induction step in the proof that all three definitions of linking number are the same.

Exercise #5: Draw a picture of a 0-framed left-handed trefoil. (Don't just write 0.)

Exercise #6: (Example 4.4.2) Consider the manifold obtained from attaching a 2-handle to a D^4 along an unknot, K. Let S be the 2-sphere created by a disk bound by K pushed into the interior of D^4 and the core of the 2-handle. Let S' be the 2-sphere created as follows. Take a noncore disk $D^2 \times p$ in the 2-handle which intersects ∂D^4 on K' a push off of K. Glue this disk to an annulus $K' \times I$ which extends into the interior of D^4. Finally, cap off the other end of the annulus with a disk in the interior. Calculate $S \cdot S'$.

Exercise #7: Let X be a 4-manifold constructed with one 0-handle and m 2-handles attached along knots $K_1, \ldots, K_m$ with framings $n_1, \ldots, n_m$.

a) What 3-manifold is ∂X?

b) (Proposition 4.5.11) Show that Q_X with respect to some basis of $H_2(X; \mathbb{Z})$ is equal to the linking pairing of $K_1, \ldots, K_m$.

c) Show that Q_X is equal to the presentation matrix of $H_1(\partial X; \mathbb{Z})$.