

MATH 206A: SYMMETRIC FUNCTIONS

HOMEWORK #7

- The homework is due on Gradescope on *Wednesday, November 23rd at 4pm*. Late homework is generally not accepted (unless you have a good reason). *This is our last homework.*
- The lowest homework score will be dropped.
- Each problem is worth the same number of points.
- Collaboration is encouraged, but you have to write up the solutions by yourself. For each problem, all sources and collaborators must be clearly listed.
- L^AT_EX is much preferred (hand-drawn pictures may be scanned). Alternatively, please submit good quality scans of your work.
- Justify your answers by rigorous proofs.

Problem 1. Show that the local rules for growth diagrams compute the jeu de taquin tableau $\text{jdt}(T)$ for $T \in \text{SYT}(\lambda/\mu)$.

Hint: [Sta99, Proposition A.1.2.7]

Problem 2. Let $w = (w_1, w_2, \dots, w_n) \in \mathfrak{S}_n$ (in one-line notation). Recall from Homework #3 that we set

$$w' := (w_n, \dots, w_2, w_1) \quad w'' := (n+1-w_n, \dots, n+1-w_2, n+1-w_1).$$

Show that if $w \xrightarrow{RSK} (P, W)$ then

$$w' \xrightarrow{RSK} (P^T, \text{evac}(Q)^T) \quad \text{and} \quad w'' \xrightarrow{RSK} (\text{evac}(P), \text{evac}(Q)).$$

Hint: [Sta99, Theorem A.1.2.10]

Problem 3. Choose a random partition λ of $n \geq 7$, and choose some μ, ν such that the Littlewood–Richardson coefficient $c_{\mu\nu}^\lambda$ is at least 3. (E.g., use your computer for that.)

- (1) Draw all tableaux T satisfying the conditions of [Sta99, Theorem A1.3.1]: that is, fix some random $P \in \text{SYT}(\nu)$, and draw all $T \in \text{SYT}(\lambda/\mu)$ that are jdt-equivalent to P .
- (2) Draw all tableaux T satisfying the conditions of [Sta99, Theorem A1.3.3]: that is, draw all $T \in \text{SSYT}(\lambda/\mu)$ of type ν whose reverse reading word is a lattice word.

Problem 4. Deduce the Pieri rule from the Littlewood–Richardson rule. Recall that the Pieri rule states that

$$h_n s_\nu = \sum_{\lambda} s_\lambda,$$

where the sum is taken over all $\lambda \supset \nu$ such that λ/ν is a horizontal n -strip.

REFERENCES

- [Sta99] Richard P. Stanley. *Enumerative combinatorics. Vol. 2*, volume 62 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1999.