

**MATH 206A: SYMMETRIC FUNCTIONS
HOMEWORK #6**

- The homework is due on Gradescope on *Monday, November 14th at 4pm*. Late homework is generally not accepted (unless you have a good reason).
- The lowest homework score will be dropped.
- Each problem is worth the same number of points.
- Collaboration is encouraged, but you have to write up the solutions by yourself. For each problem, all sources and collaborators must be clearly listed.
- L^AT_EX is much preferred (hand-drawn pictures may be scanned). Alternatively, please submit good quality scans of your work.
- Justify your answers by rigorous proofs.

Problem 1. Use the Lindström-Gessel-Viennot lemma to show directly that

$$s_{\lambda/\mu} = \det(e^{\lambda'_i - \mu'_j - i + j})_{i,j=1}^n.$$

Problem 2. Show that if $u, v \in \mathfrak{S}_n$ are Knuth-equivalent then $I_k(u) = I_k(v)$ for all k .

Hint: [Sta99, Lemma A.1.1.7]

Problem 3. Let $P \in \text{SYT}(\lambda)$ and $w = \text{read}(P)$. Compute $I_k(w)$ in terms of λ .

Hint: [Sta99, Proof of Theorem A.1.1.1]

REFERENCES

- [Sta99] Richard P. Stanley. *Enumerative combinatorics. Vol. 2*, volume 62 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1999.