MATH 206A: SYMMETRIC FUNCTIONS
HOMEWORK #3

• The homework is due on Gradescope on *Monday, October 24th at 4pm*. Late homework is generally not accepted (unless you have a good reason).
• The lowest homework score will be dropped.
• Each problem is worth the same number of points.
• Collaboration is encouraged, but you have to write up the solutions by yourself. For each problem, all sources and collaborators must be clearly listed.
• LaTeX is much preferred (hand-drawn pictures may be scanned). Alternatively, please submit good quality scans of your work.
• Justify your answers by rigorous proofs.

Problem 1. Let $(a_{\lambda\mu})_{\lambda,\mu\in\operatorname{Par}(n)}$ and $(b_{\lambda\mu})_{\lambda,\mu\in\operatorname{Par}(n)}$ be the matrices given by

$$a_{\lambda\mu} := \langle s_\lambda, h_\mu \rangle, \quad e_\lambda = \sum_\mu b_{\lambda\mu} s_{\mu'}.$$

Added on 10/20/22: Here μ' is the conjugate partition of μ. Choose some $n \geq 13$ and a random[1] three-element subset $X \subset \operatorname{Par}(n)$. Turn in a list of three partitions in X and two 3×3 matrices $(a_{\lambda\mu})_{\lambda,\mu\in X}$ and $(b_{\lambda\mu})_{\lambda,\mu\in X}$. No justification necessary.

Hint: Use your computer.

Problem 2. Choose $n \geq 10$ and a random[2] permutation $w = (w_1, w_2, \ldots, w_n) \in \mathfrak{S}_n$. Let

$$w' := (w_n, \ldots, w_2, w_1) \quad w'' := (n+1-w_n, \ldots, n+1-w_2, n+1-w_1).$$

Compute the images of w, w', w'' under the RSK algorithm. (Again, different people should submit different permutations.) No justification necessary.

In addition, solve the following exercises from [Sta99]**:**

- Exercise 7.8.
- Exercise 7.11.
- Exercise 7.12.

References

[Sta99] Richard P. Stanley. *Enumerative combinatorics. Vol. 2*, volume 62 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1999.

Date: October 20, 2022.

[1]There are 101 partitions of 13, so about a million possible choices of X. I will take off points whenever two people choose identical sets X.

[2]Note that 10! is about 3.6 million.