MATH 206A: SYMMETRIC FUNCTIONS
HOMEWORK #2

• The homework is due on Gradescope on *Monday, October 17th at 4pm*. Late homework is generally not accepted (unless you have a good reason).
• Upd: The lowest homework score will be dropped.
• Each problem is worth the same number of points.
• Collaboration is encouraged, but you have to write up the solutions by yourself. For each problem, all sources and collaborators must be clearly listed.
• LaTeX is much preferred (hand-drawn pictures may be scanned). Alternatively, please submit good quality scans of your work.
• Justify your answers by rigorous proofs.

Problem 1.

(1) Using the Exponential formula for generating functions, find the number $h(n)$ of ways to partition a set of n people into groups and then have each group sit around a circular table.
(2) Find the number $h'(n)$ of ways to partition a set of n people into groups, have each group sit around a circular table, and serve either red or white wine to each table.

Optional: It is easy to find a formula for $h(n)$ by combinatorial reasoning without using generating functions. Try doing the same for $h'(n)$.

Hint: [Bóna16, Examples 3.31 and 3.37]

Problem 2. Show that

$$h_n = \sum_{\lambda \vdash n} z_\lambda^{-1} p_\lambda \quad \text{and} \quad e_n = \sum_{\lambda \vdash n} \varepsilon_\lambda z_\lambda^{-1} p_\lambda.$$

Hint: [Sta99, Prop. 7.7.6]

In addition, solve the following exercises from [Sta99]**:**

- Exercise 7.5.
- Exercise 7.7.
- Exercise 7.9.

References

[Bóna16] Miklós Bóna. *Introduction to enumerative and analytic combinatorics*. Discrete Mathematics and its Applications (Boca Raton). CRC Press, Boca Raton, FL, second edition, 2016.

[Sta99] Richard P. Stanley. *Enumerative combinatorics. Vol. 2*, volume 62 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1999.

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