

## MATH 206 CLUSTER ALGEBRAS: HOMEWORK #1

- The homework is due on Gradescope on *Wednesday, January 27th at 1pm*. Late homework is generally not accepted (unless you have a good reason).
- Each problem is worth the same number of points.
- Collaboration is encouraged, but you have to write up the solutions by yourself. For each problem, all sources and collaborators must be clearly listed.
- $\text{\LaTeX}$  is preferred (hand-drawn pictures may be scanned). Alternatively, please submit good quality scans of your work! (e.g. google “phone scan app”)
- Justify your answers by rigorous proofs.
- The total number of problems you turn in should be *five* (or less).

### 1. MANDATORY PROBLEMS

Solve *both* of these problems and turn them in. All references are to the book [FWZ16] by Fomin–Williams–Zelevinsky cited below.

1. [FWZ16, Exercise 2.6.8] (“Dynkin diagrams”)
2. [FWZ16, Exercise 3.4.7] (Somos-4 sequence)

**UPD:** The Somos-4 exercise in the book asks to show that the coefficients lie in  $\mathbb{Z}[a, b]$ . For this homework, it is enough to show that they lie in  $\mathbb{Z}[a^{\pm 1}, b^{\pm 1}]$ . Alternatively, for  $\mathbb{Z}[a, b]$ , you can use [FWZ16, Theorem 3.3.6] which I didn’t mention in class.

### 2. OPTIONAL PROBLEMS

Solve all of the below problems, but turn in exactly *three* of them which you found the most interesting. All exercises that were mentioned (or could have been mentioned) in class are also considered to be members of this list.

3. [FWZ16, Exercise 1.2.2] (3-term Plücker relations for  $2 \times n$  matrices)
4. [FWZ16, Exercise 1.3.4] (relations for flag minors)
5. [FWZ16, Exercise 2.1.4] (properties of quiver mutations)
6. [FWZ16, Exercise 2.2.2] (flips vs mutations)
7. [FWZ16, Exercise 2.3.3] (braid moves vs mutations)
8. [FWZ16, Exercise 2.6.5] (orientations of a tree)
9. [FWZ16, Exercise 2.6.6] (orientations of an  $n$ -cycle)
10. [FWZ16, Exercise 2.6.7] (grid quiver)
11. [FWZ16, Exercise 2.6.9] (triangular grid quiver)
12. [FWZ16, Exercise 3.4.6] (another sequence)
13. [FWZ16, Exercise 3.4.9] (Somos-5 sequence)

### REFERENCES

[FWZ16] Sergey Fomin, Lauren Williams, and Andrei Zelevinsky. Introduction to Cluster Algebras. Chapters 1-3. [arXiv:1608.05735v3](https://arxiv.org/abs/1608.05735v3), 2016.