Zamolodchikov periodicity and integrability

Pavel Galashin

MIT

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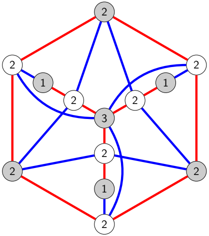
UCLA, October 26, 2018

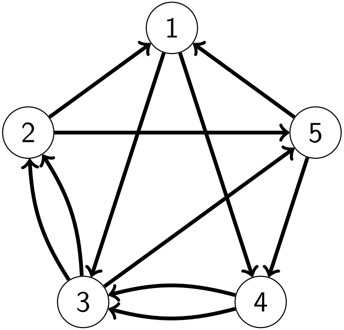
Joint work with Pavlo Pylyavskyy



Part 1: T-systems

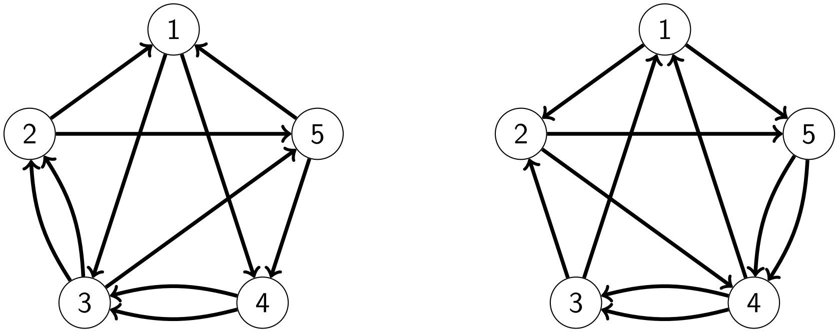
General T-systems (Nakanishi, 2011)

Q

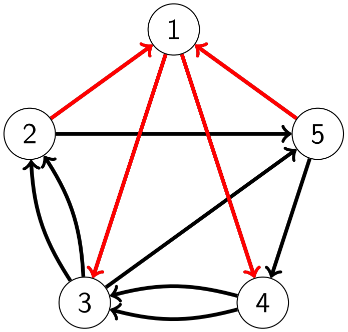


General T-systems (Nakanishi, 2011)

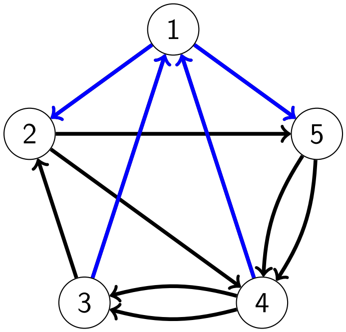
Q $\qquad\qquad$ $\mu_1(Q)$



Q

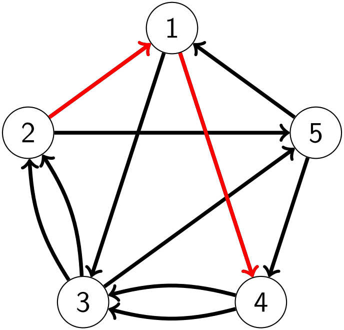


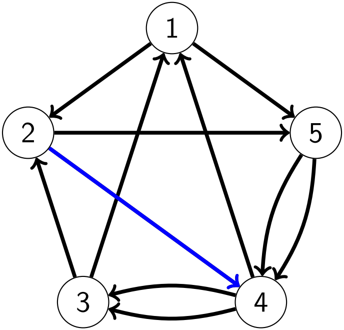
$\mu_1(Q)$



General T-systems (Nakanishi, 2011)

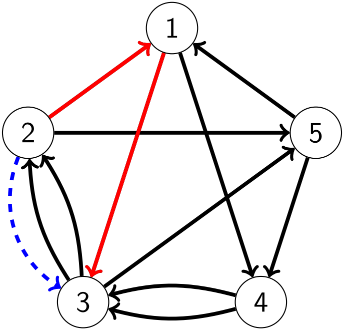
Q $\mu_1(Q)$



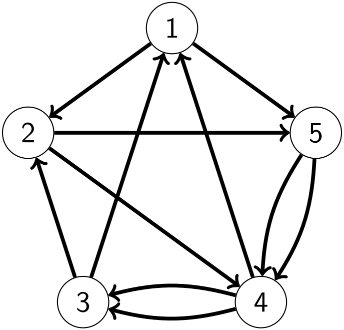


General T-systems (Nakanishi, 2011)

Q

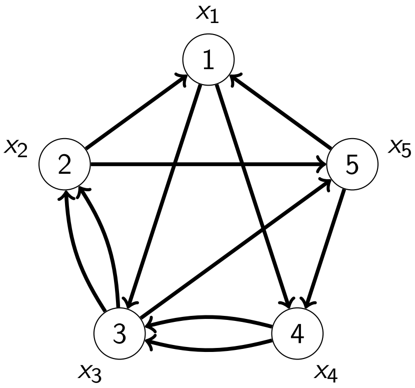


$\mu_1(Q)$

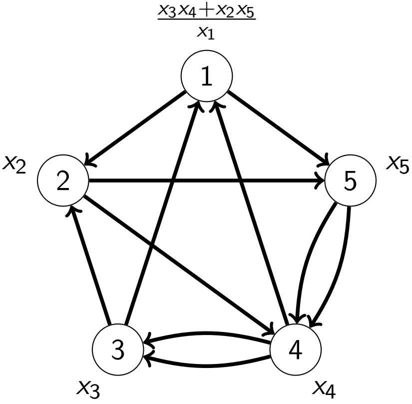


General T-systems (Nakanishi, 2011)

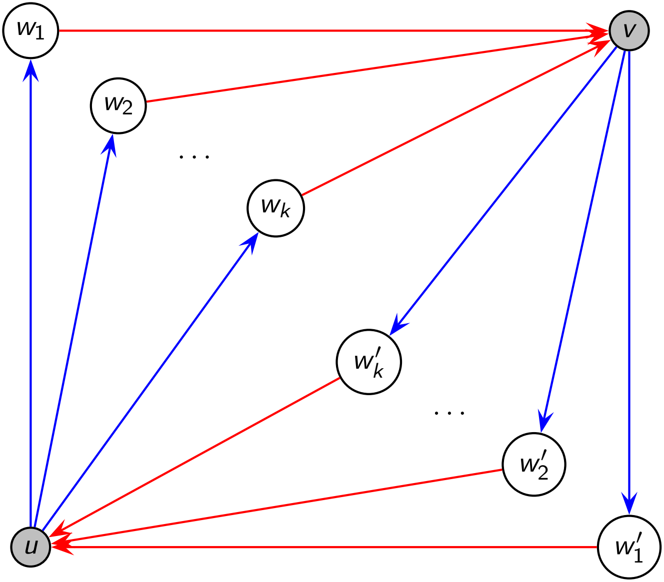
Q



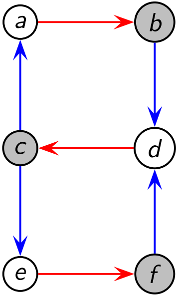
$\mu_1(Q)$



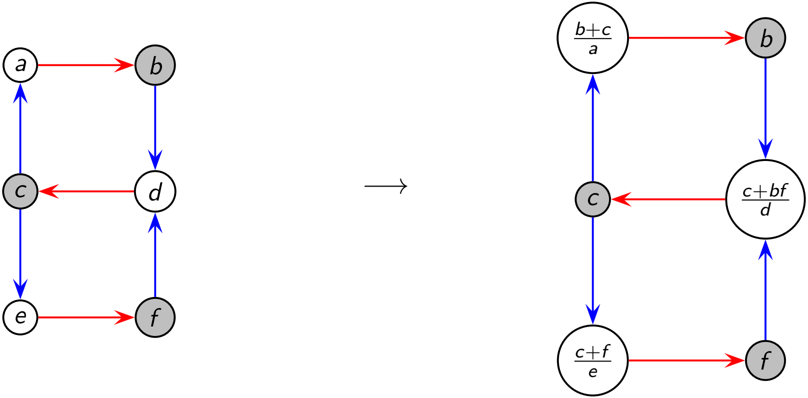
Bipartite **recurrent** quivers



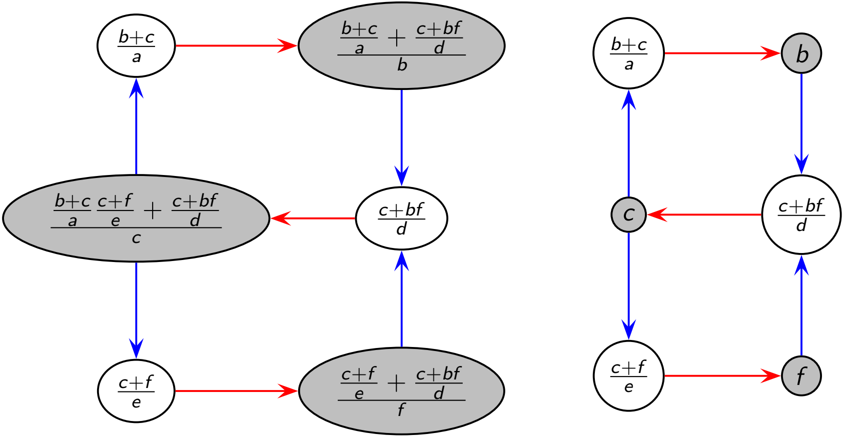
Bipartite T-system



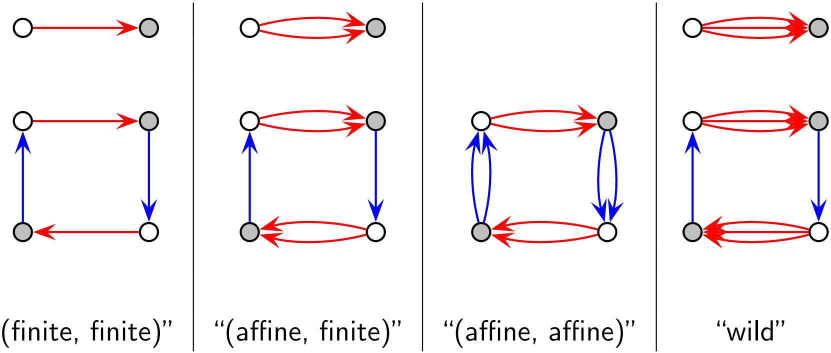
Bipartite T-system



Bipartite T-system

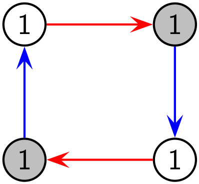


Four classes of quivers

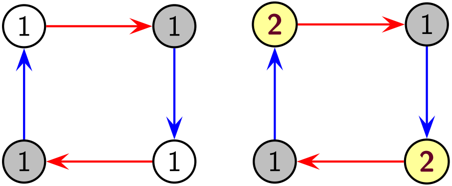


“(finite, finite)” | “(affine, finite)” | “(affine, affine)” | “wild”

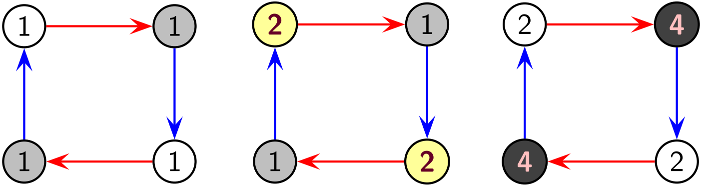
Example: (finite, finite)



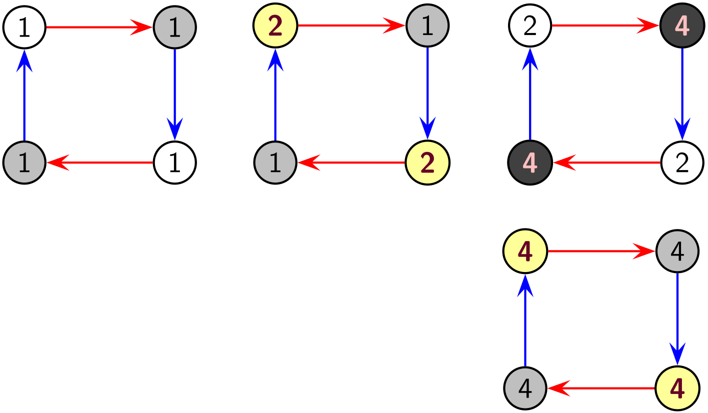
Example: (finite, finite)



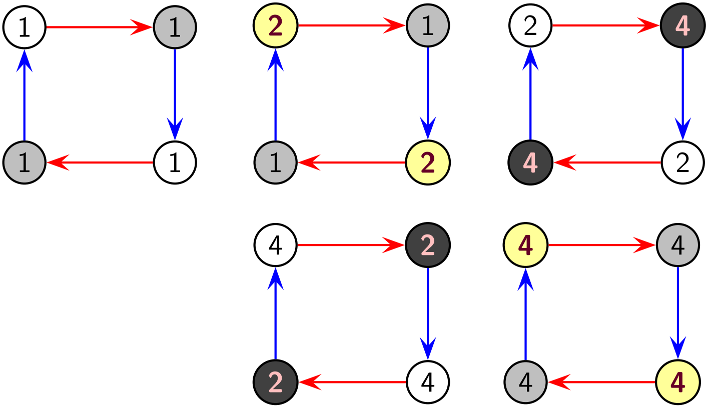
Example: (finite, finite)



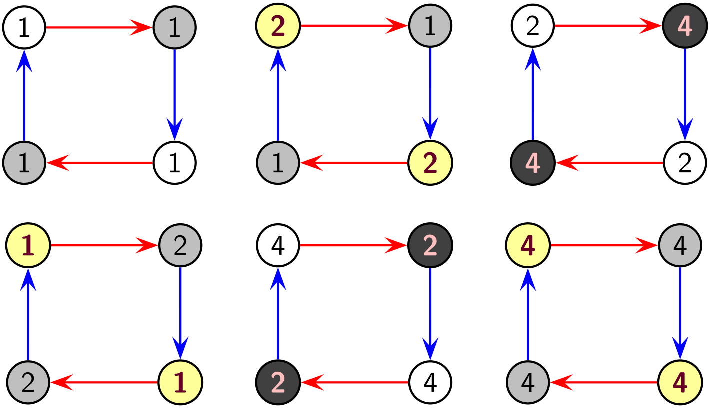
Example: (finite, finite)



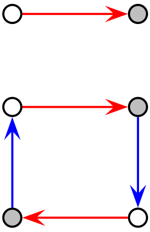
Example: (finite, finite)



Example: (finite, finite)

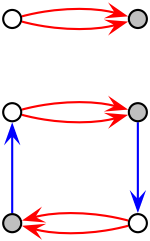


Four classes of quivers

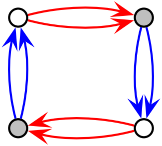


“(finite, finite)”

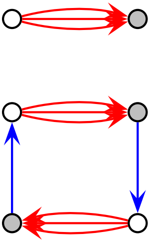
periodic



“(affine, finite)”



“(affine, affine)”



“wild”

Example: (affine, finite)

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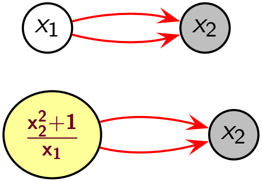

Example: (affine, finite)

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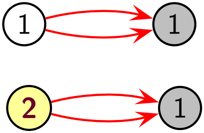
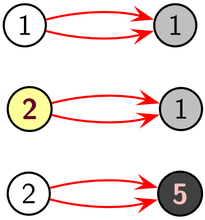
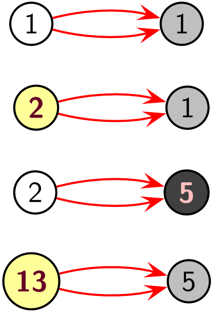


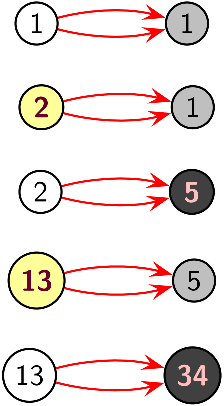
Example: (affine, finite)



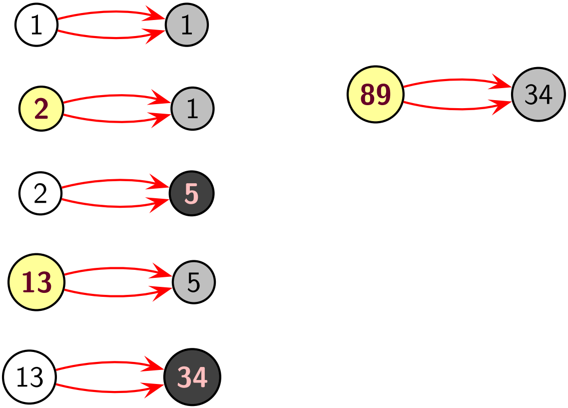
Example: (affine, finite)



Example: (affine, finite)



Example: (affine, finite)



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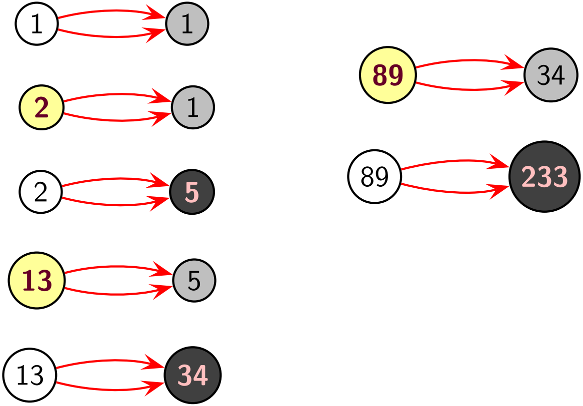
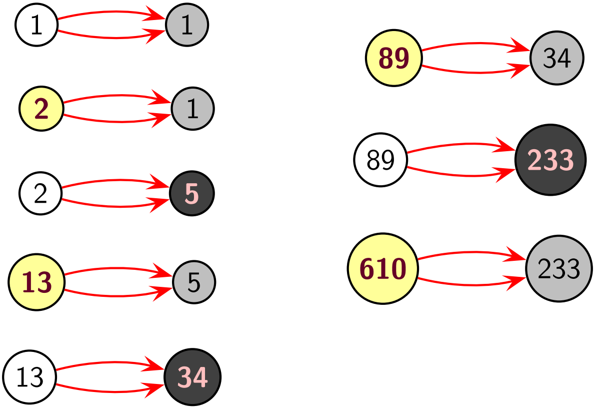
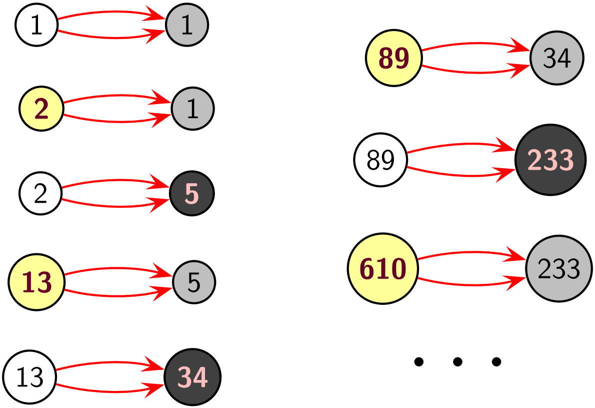


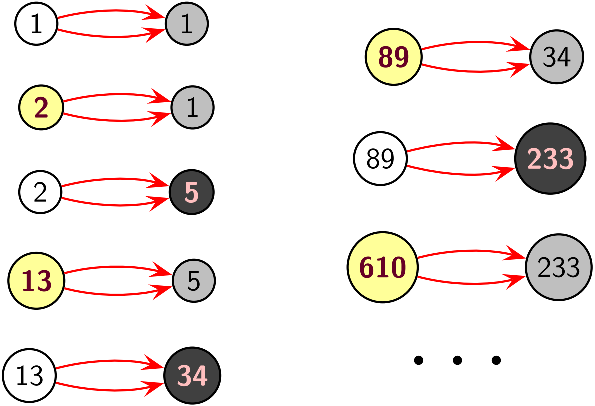
Example: (affine, finite)



Example: (affine, finite)

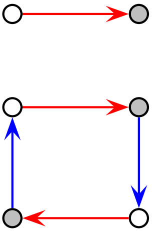


Example: (affine, finite)



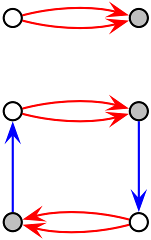
$$x_{n+1} = 3x_n - x_{n-1}$$

Four classes of quivers



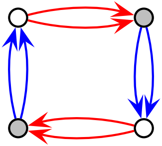
"(finite, finite)"

periodic

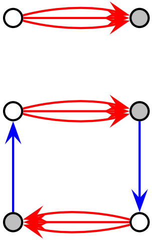


"(affine, finite)"

linearizable

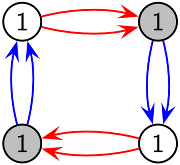


"(affine, affine)"

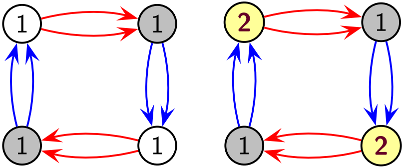


"wild"

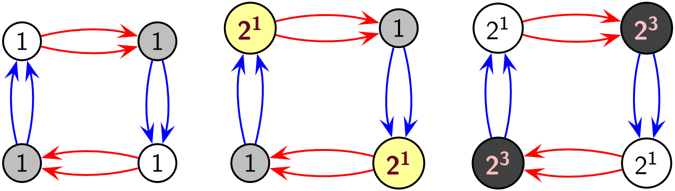
Example: (affine, affine)



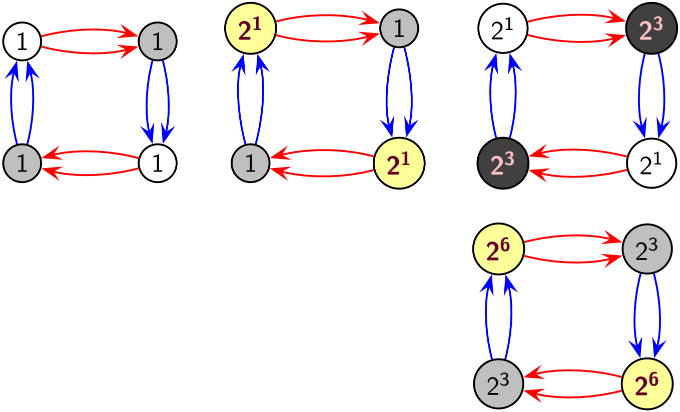
Example: (affine, affine)



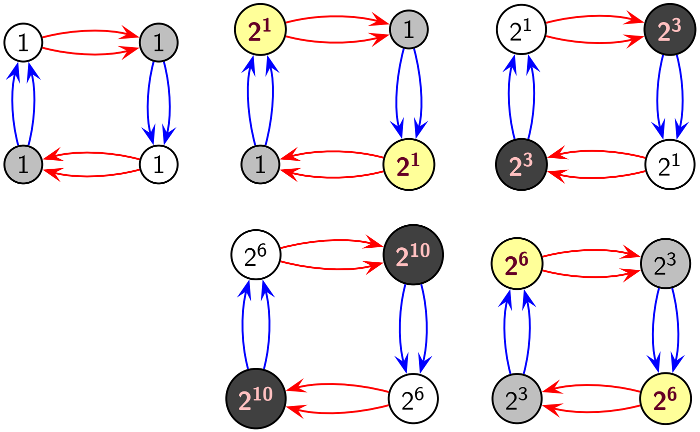
Example: (affine, affine)



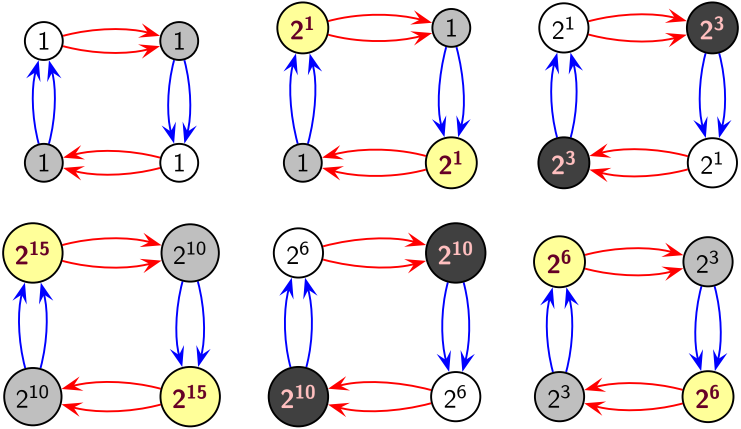
Example: (affine, affine)



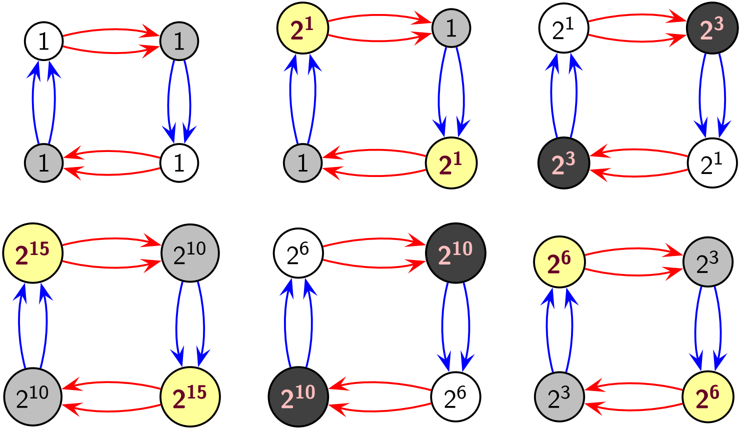
Example: (affine, affine)



Example: (affine, affine)

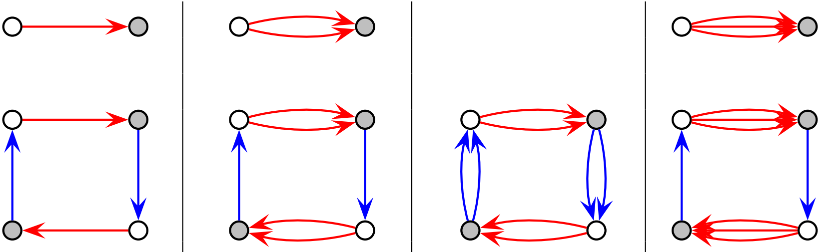


Example: (affine, affine)



$$2^{\binom{n}{2}}$$

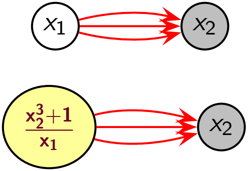
Four classes of quivers



"(finite, finite)"	"(affine, finite)"	"(affine, affine)"	"wild"
periodic	linearizable	**grows as** $\exp(t^2)$	

Example: wild

Example: wild



Example: wild

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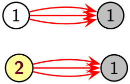
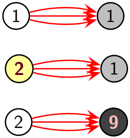
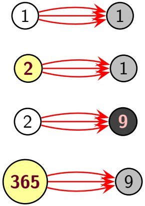


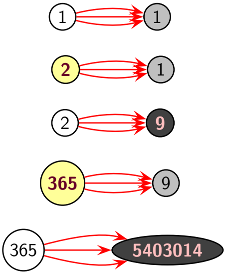
Example: wild



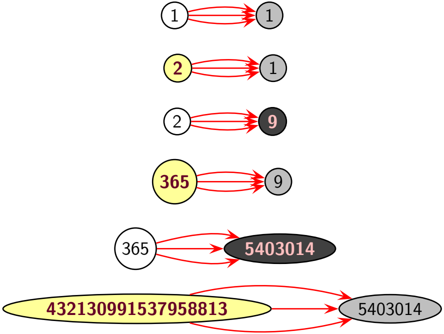
Example: wild



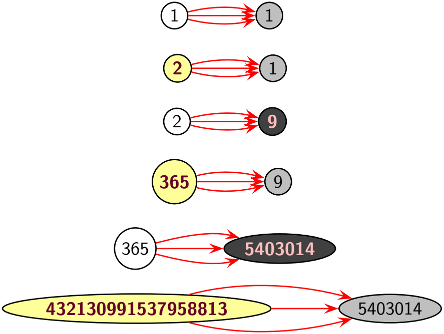
Example: wild



Example: wild



Example: wild



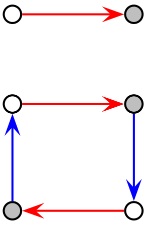
A003818 a(1)=a(2)=1, a(n+1) = (a(n)^3 +1)/a(n-1).

1, 1, 2, 9, 365, 5403014, 4321309915379588l3,
149351692841015258744916734632684145365235930 57 (list; graph; refs; lis

OFFSET 1,3

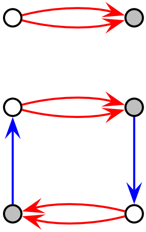
COMMENTS The term a(9) has 121 digits. - Harvey P. Dale,

Four classes of quivers



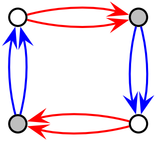
“(finite, finite)”

periodic



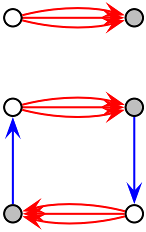
“(affine, finite)”

linearizable



“(affine, affine)”

grows as $\exp(t^2)$

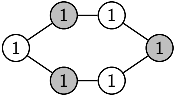
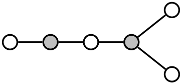
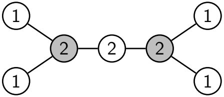
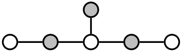
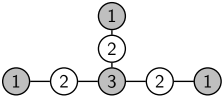
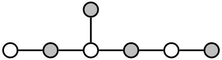
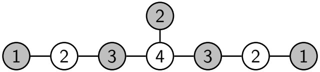
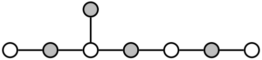
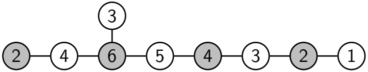


“wild”

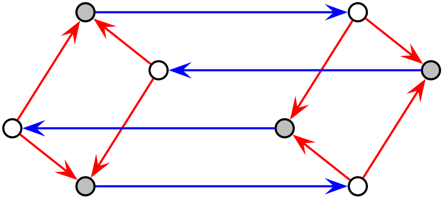
grows as $\exp(\exp(t))$

Part 2: The master conjecture

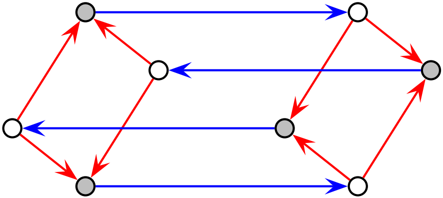
ADE Dynkin diagrams

Name	Finite diagram	Affine diagram	Name
A_n			$\hat{A}_{n-1}$
D_n			$\hat{D}_{n-1}$
E_6			$\hat{E}_6$
E_7			$\hat{E}_7$
E_8			$\hat{E}_8$

(affine, finite) quivers

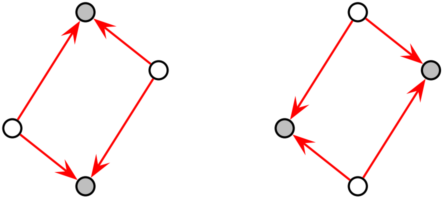


(affine, finite) quivers



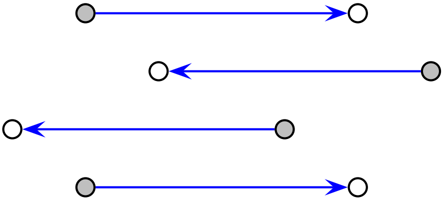
- Bipartite recurrent quiver

(affine, finite) quivers



- Bipartite recurrent quiver
- All red components are **affine** Dynkin diagrams

(affine, finite) quivers



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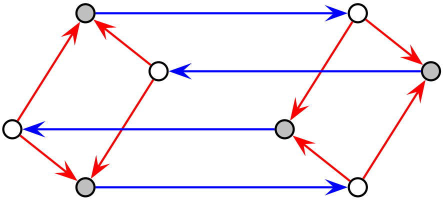
- All blue components are **finite** Dynkin diagrams

(affine, finite) quivers



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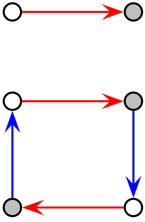
- All red components are **affine** Dynkin diagrams
- All blue components are **finite** Dynkin diagrams

↑

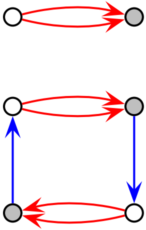
"**(affine, finite) quiver**"

Four classes of quivers



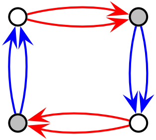
"(finite, finite)"

periodic



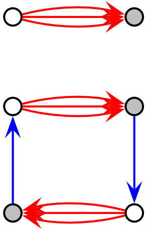
"(affine, finite)"

linearizable



"(affine, affine)"

grows as $\exp(t^2)$



"wild"

grows as $\exp(\exp(t))$

Master conjecture

Conjecture (G.-Pylyavskyy, 2016)

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- *(finite, finite)* $\Longleftrightarrow$ *periodic*

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- *wild* $\iff$ *grows as* $\exp(\exp(t))$

Results

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Periodic $\iff$ *(finite, finite)*

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Grows slower than $\exp(\exp(t)) \Longrightarrow$ *(affine, affine), (affine, finite), or (finite, finite)*

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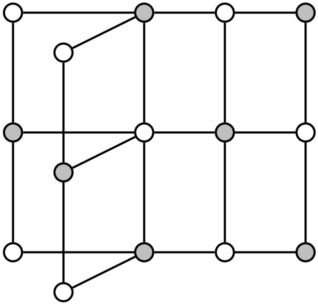
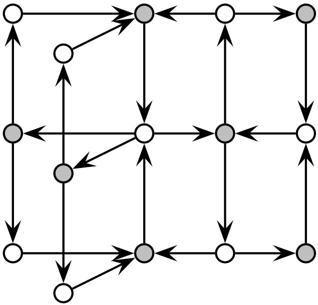
- *(affine, finite)* $\implies$ *linearizable*
- *(affine, affine)* $\implies$ *grows as* $\exp(t^2)$

Tensor product



$D_5 \otimes A_3$

Tensor product



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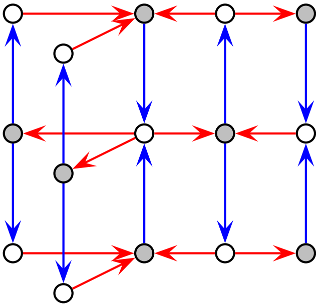

$D_5 \otimes A_3$

Zamolodchikov periodicity

Theorem (B. Keller, 2013)

Tensor product of **finite** *Dynkin diagrams* $\Longrightarrow$ *the T-system is periodic.*

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History of cluster algebras

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Zamolodchikov periodicity conjecture holds for $\Lambda \otimes A_1$.

Proof.

Use the above bijection and then prove periodicity for the tropical dynamics on $\Phi_{\geq -1}$. □

Example: A_2

Example: A_2

x_1

Example: A_2

x_1

x_2

Example: A_2

$$x_1 \qquad \qquad x_2$$

$$\frac{x_2+1}{x_1}$$

Example: A_2

x_1

x_2

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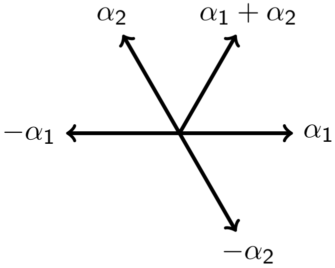
$$\frac{x_1+1}{x_2}$$

$$x_1$$

$$x_2$$

$$\ldots$$

Example: A_2



x_1

x_2

$$\frac{x_2 + 1}{x_1}$$

$$\frac{x_1 + x_2 + 1}{x_1 x_2}$$

$$\frac{x_1 + 1}{x_2}$$

x_1

x_2

$\dots$

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Tensor product of **finite** *Dynkin diagrams* $\Longrightarrow$ *the* T*-system is periodic.*

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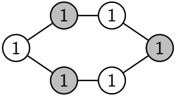
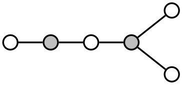
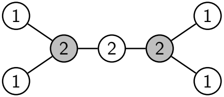
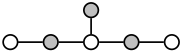
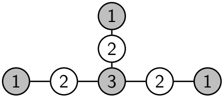
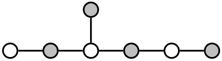
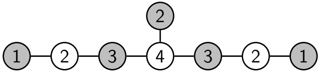
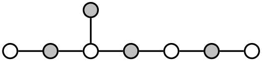
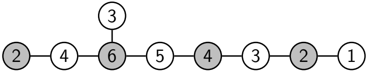
ADE Dynkin diagrams

Name	Finite diagram	h	Affine diagram	Name
A_n		$\mathbf{n+1}$		$\hat{A}_{n-1}$
D_n		$\mathbf{2n-2}$		$\hat{D}_{n-1}$
E_6		**12**		$\hat{E}_6$
E_7		**18**		$\hat{E}_7$
E_8		**30**		$\hat{E}_8$

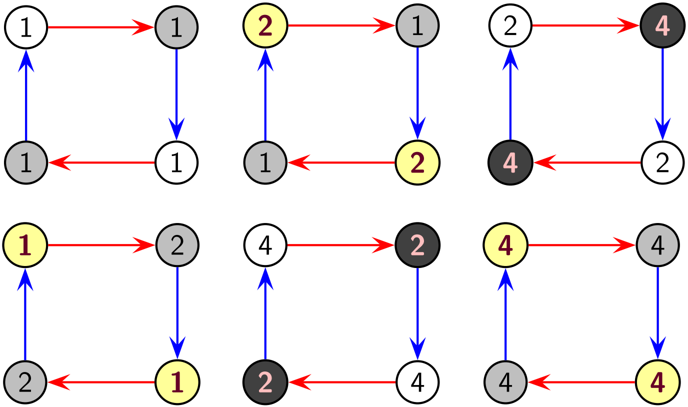
Zamolodchikov periodicity

Theorem (B. Keller, 2013)

Tensor product of **finite** *Dynkin diagrams* $\Longrightarrow$ *the T-system is periodic* **with period dividing**

$$2(h + h').$$

Example: $A_2 \otimes A_2$



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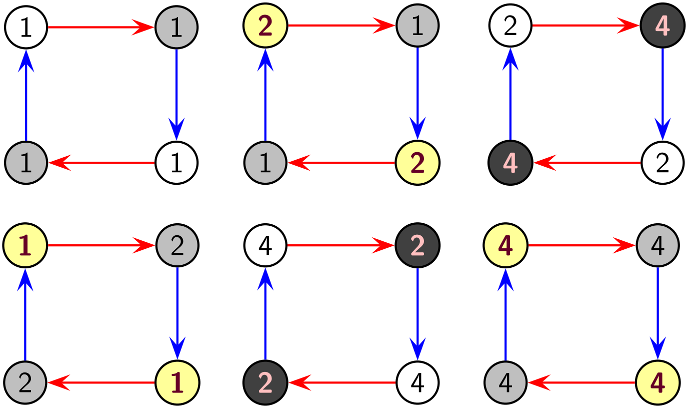

6 steps!

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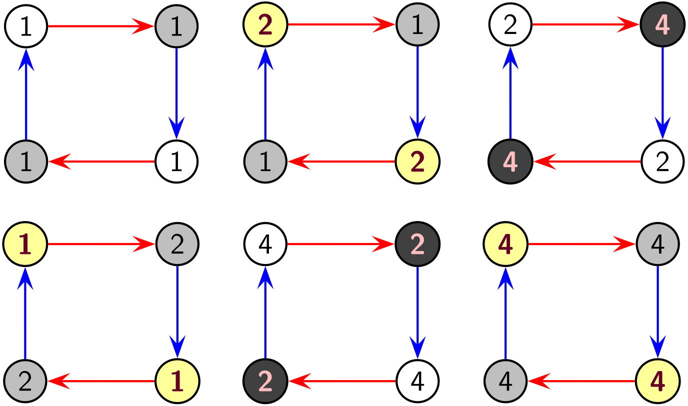

~~6~~ 12 **steps!**

Results

Theorem (G.-Pylyavskyy, 2016)

Periodic $\iff$ (finite, finite)

Theorem (G.-Pylyavskyy, 2016)

Linearizable $\implies$ *(affine, finite) or (finite, finite)*

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Grows slower than $\exp(\exp(t))$ $\implies$ *(affine, affine), (affine, finite), or (finite, finite)*

What is left:

Conjecture (G.-Pylyavskyy, 2017)

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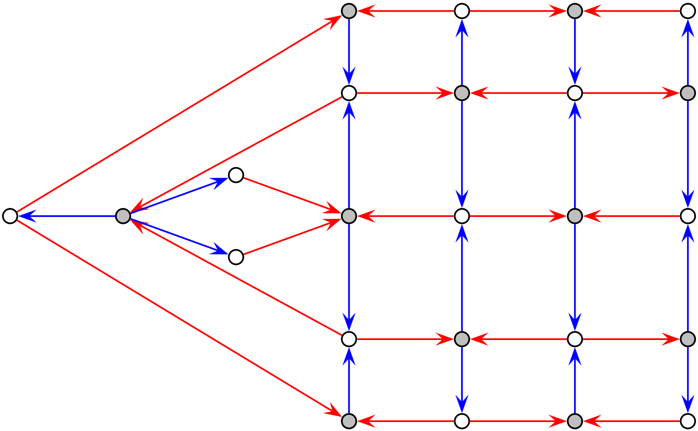
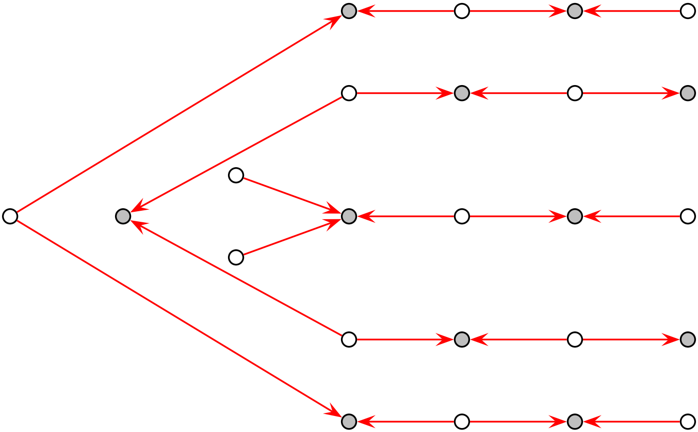


(finite, finite) quivers



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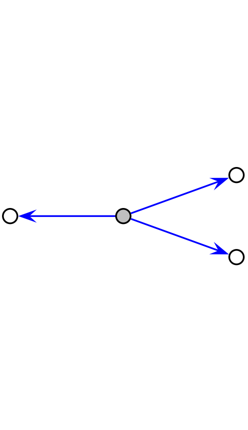
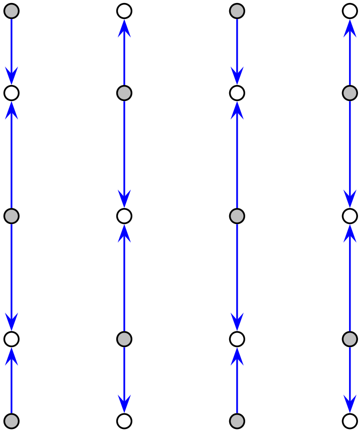
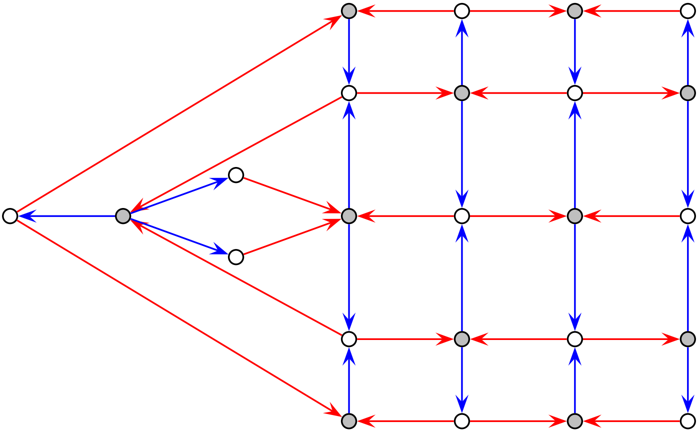




(finite, finite) quivers



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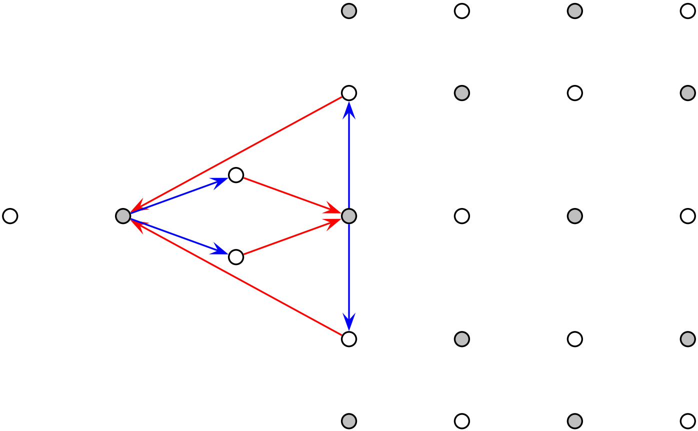

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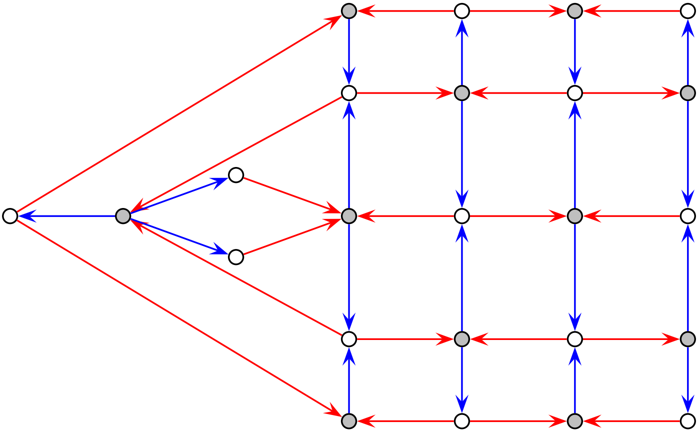

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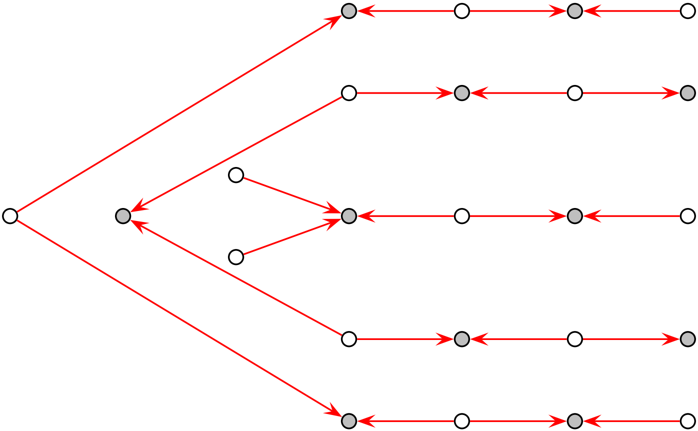
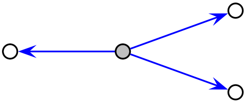
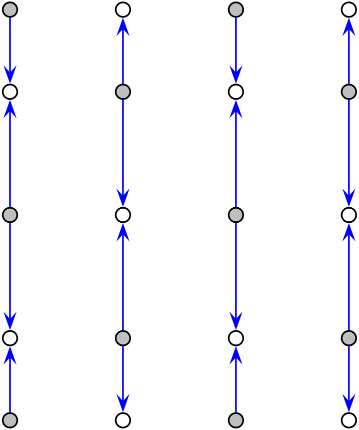


$\mathbf{h} = \mathbf{9} + \mathbf{1} = \mathbf{12} - \mathbf{2} = \mathbf{10}$;

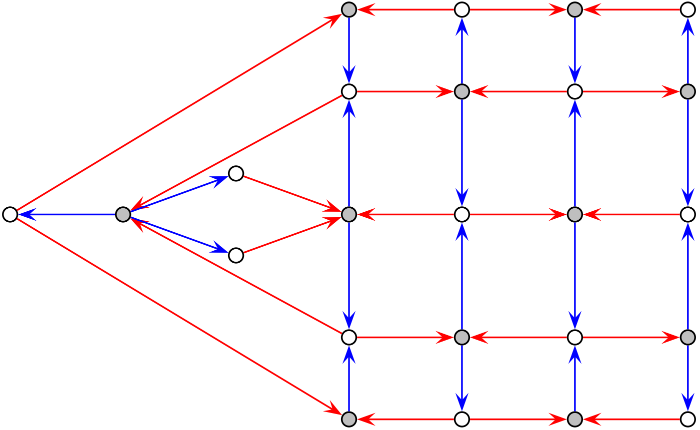
(finite, finite) quivers





$h = 9 + 1 = 12 - 2 = 10$; $\mathbf{h' = 5 + 1 = 8 - 2 = 6}$;

(finite, finite) quivers

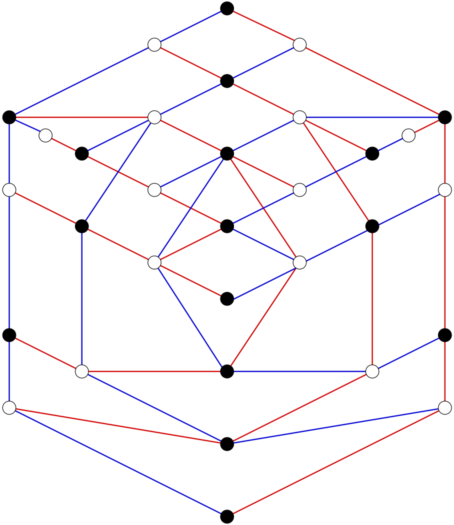


$h = 9 + 1 = 12 - 2 = 10; \quad h' = 5 + 1 = 8 - 2 = 6;$ Period = **32**

Part 3: The classification

(finite, finite) classification (Stembridge, 2010)

5 infinite families and 8 exceptional quivers



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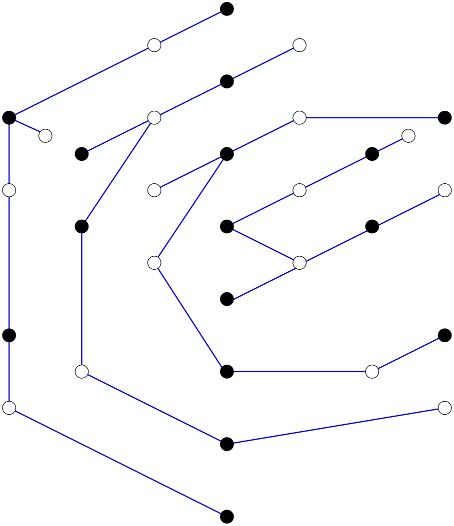

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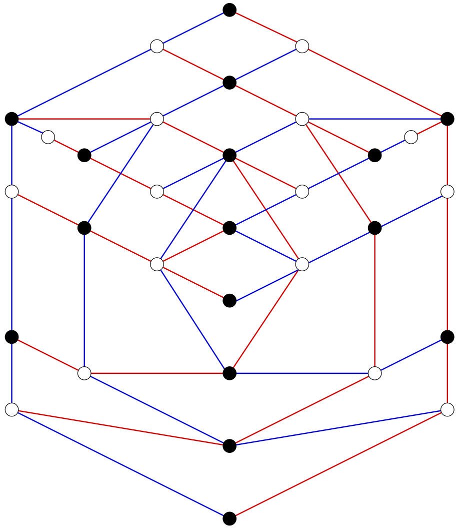
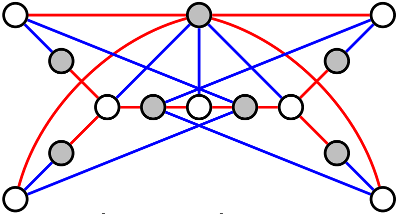
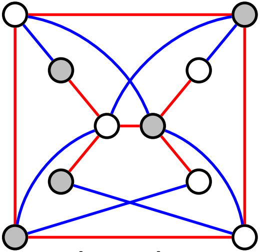


15 infinite families and 4 exceptional cases



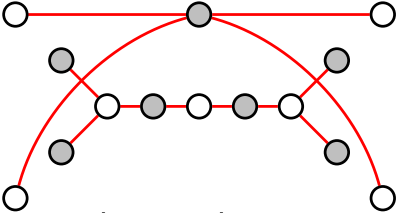
$\hat{D}_{n+1} * \hat{D}_{3n-1}$
for $n = 3$



$\hat{A}_3 * \hat{D}_5$

(affine, finite) classification

15 infinite families and 4 exceptional cases



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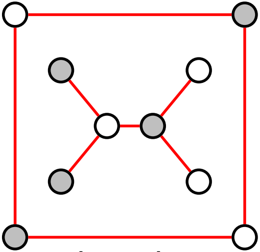

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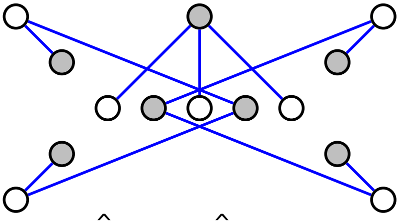

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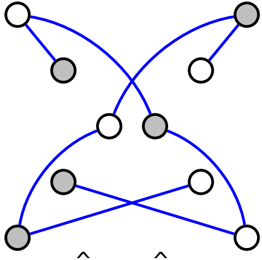

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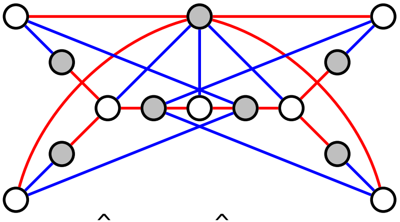

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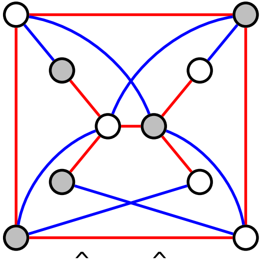

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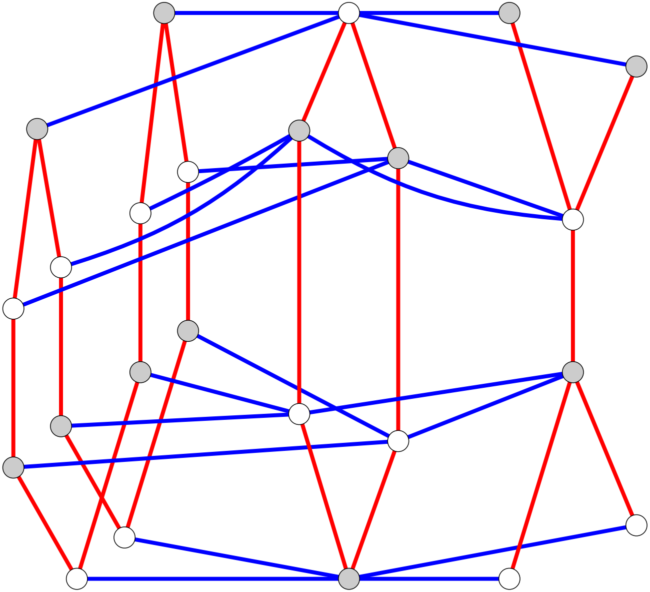
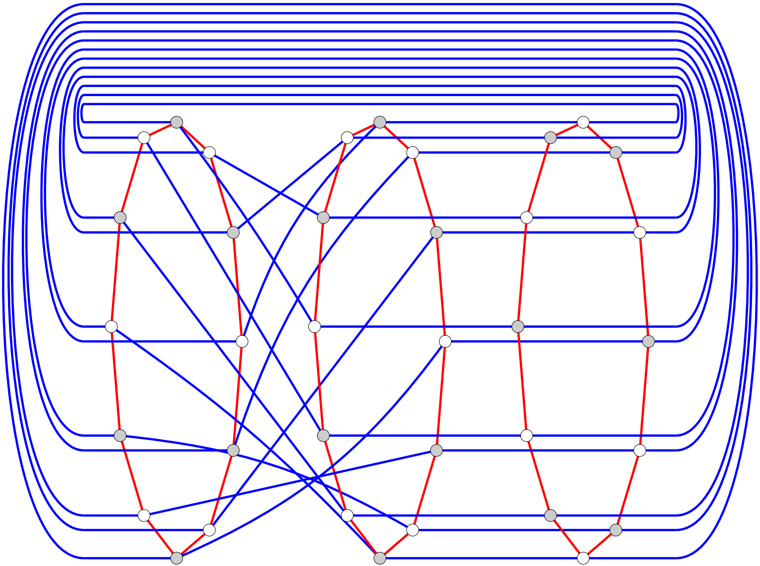
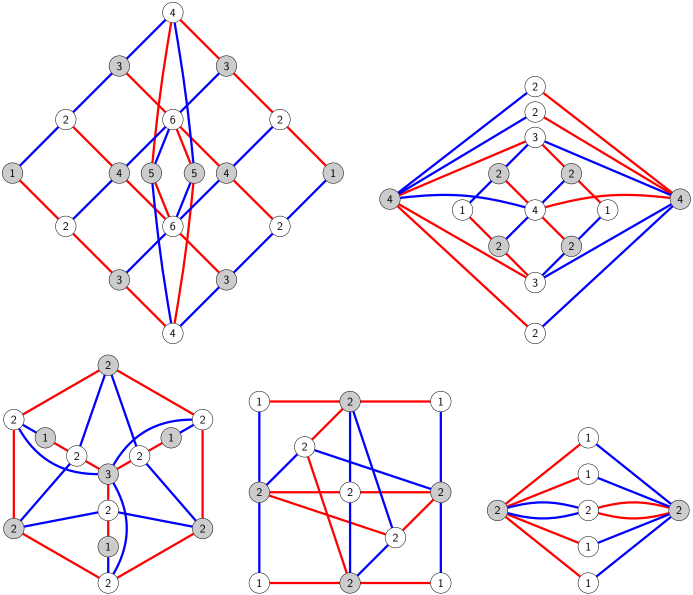
(affine, affine) quivers



Toric quivers



(affine, affine) classification: 40 infinite, 13 exceptional



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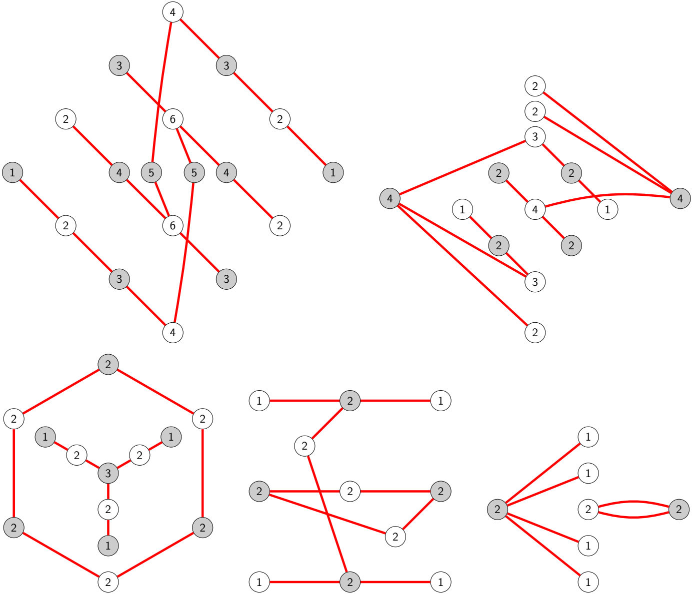

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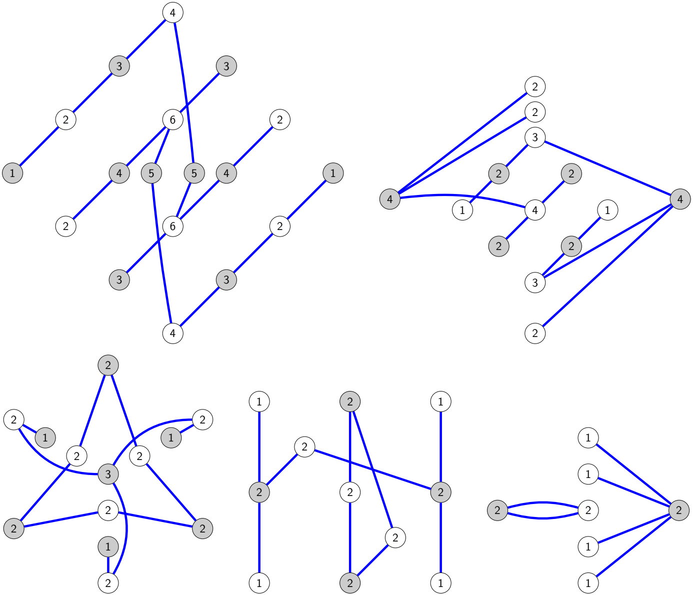

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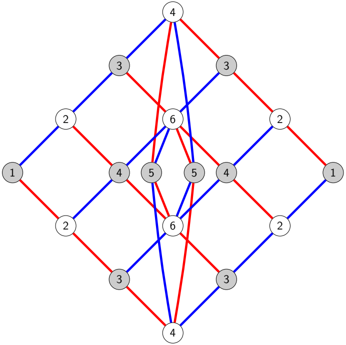

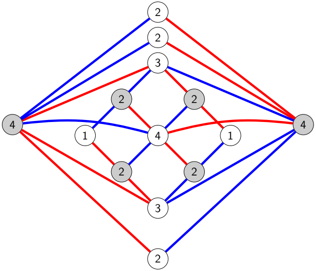

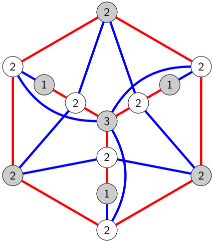

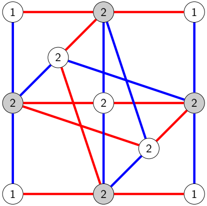

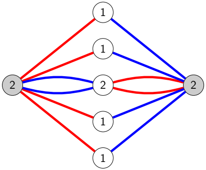

Results

Theorem (G.-Pylyavskyy, 2016)

Periodic $\iff$ *(finite, finite)*

Theorem (G.-Pylyavskyy, 2016)

Linearizable $\Longrightarrow$ *(affine, finite) or (finite, finite)*

Theorem (G.-Pylyavskyy, 2017)

Grows slower than $\exp(\exp(t)) \Longrightarrow$ **(affine, affine), (affine, finite), or (finite, finite)**

What is left:

Conjecture (G.-Pylyavskyy, 2017)

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- *(affine, affine)* $\Longrightarrow$ *grows as* $\exp(t^2)$

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Bibliography

Slides: `http://math.mit.edu/~galashin/slides/ucla_zam.pdf`

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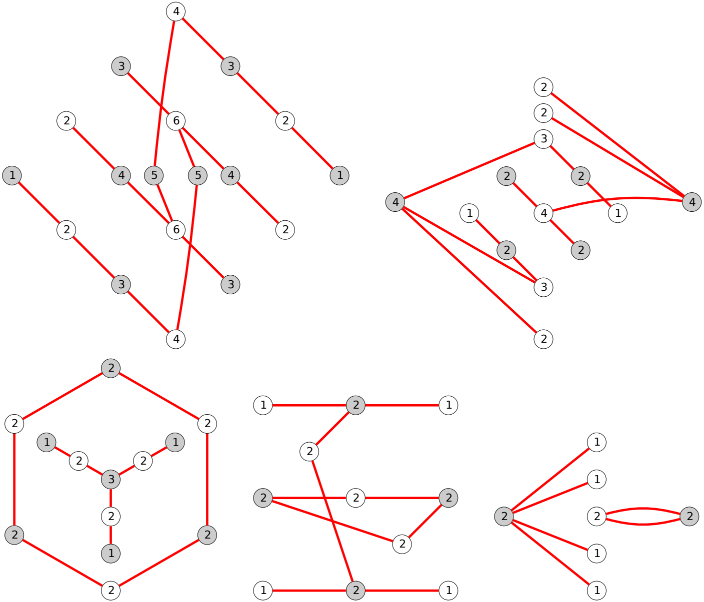
Thank you!



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