Existence of a persistent hub in the convex preferential attachment model

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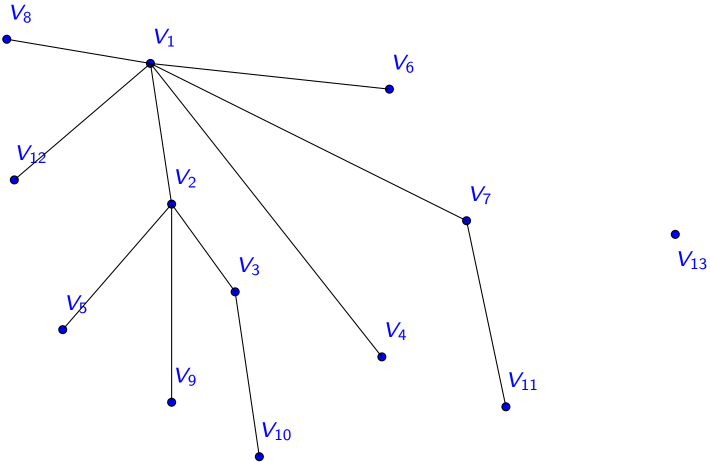
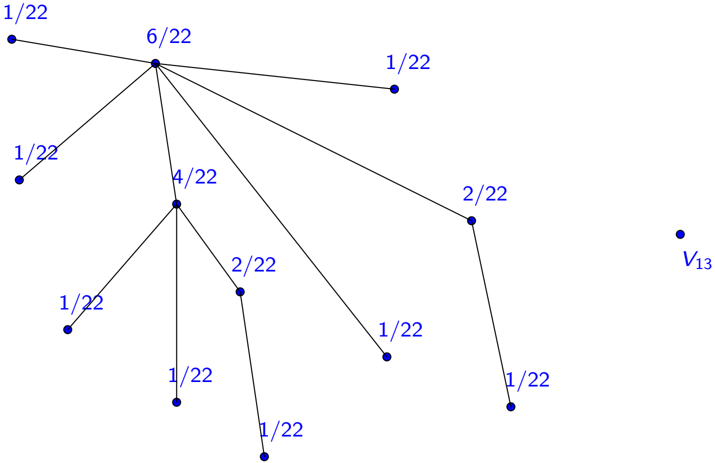
Barabási-Albert random graph



Barabási-Albert random graph



Persistent hub

Question

Is there always a vertex which has the maximal degree for all but finitely many steps?

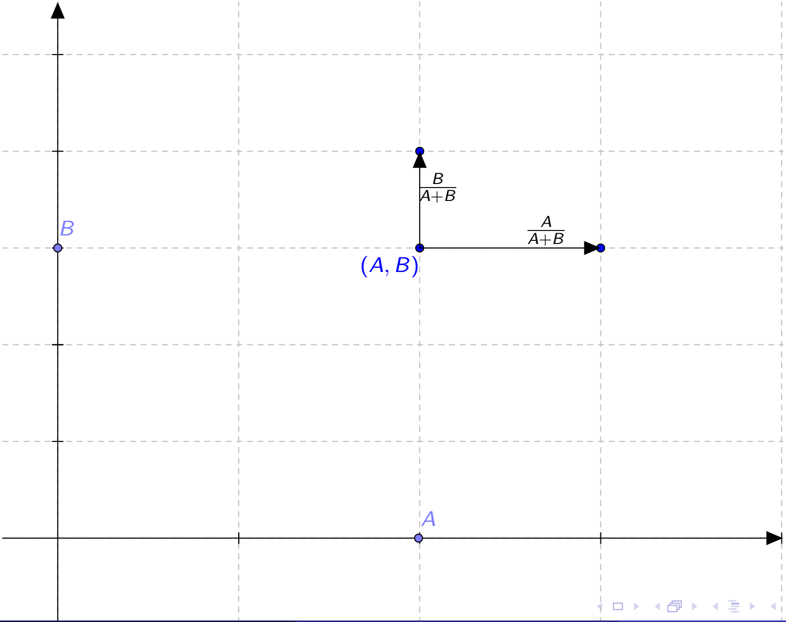
Definition

Such vertex is called *a persistent hub*.

Theorem

A persistent hub appears with probability 1.

Two-dimensional random walk



Probabilities of paths

Lemma

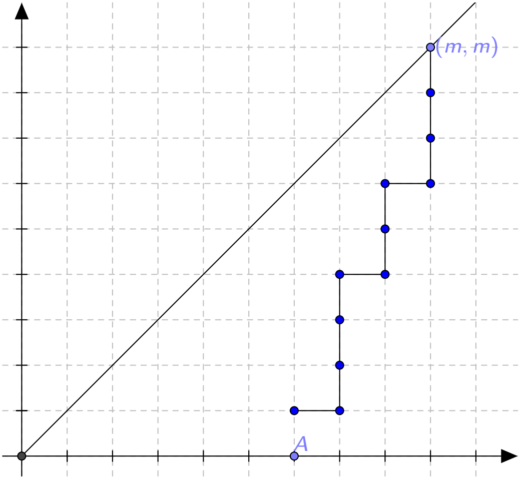
Let (A_1, B_1) *and* (A_2, B_2) *be two points. Then for any path* S *connecting these points its probability equals*

$$\mathbb{P}(S) = \frac{A_1 \cdot (A_1 + 1) \cdot \ldots \cdot (A_2 - 1) \cdot B_1 \cdot (B_1 + 1) \cdot \ldots \cdot (B_2 - 1)}{(A_1 + B_1) \cdot (A_1 + B_1 + 1) \cdot \ldots \cdot (A_2 + B_2 - 1)}.$$

Remark

This probability does not depend on S*.*

The probability of crossing the diagonal



$$\mathbb{P}[(A,1) \to \text{ diagonal }] \le P(A)/2^A.$$

Finite number of leaders

Lemma

$$\mathbb{P}[(A,1) \to \textit{ diagonal }] \leq P(A)/2^A.$$

Lemma

The degree of the first vertex after n-th step grows like $C\sqrt{n}$ for some random non-zero constant C.

Corollary

For almost all vertices their degree will always be lower than the degree of the first vertex, because the series

$$\sum_{n=1}^{\infty} \frac{P(C\sqrt{n})}{2^{C\sqrt{n}}}$$

is convergent for any C.

Pólya urn

Proposition

If our random walk starts at the point $(A, 1)$ *then the quantity* $A_k/(A_k + B_k)$ *tends to some random variable* $H(A)$ *as* k *tends to infinity. Moreover,* $H(A)$ *has a beta probability distribution:*

$$H(A) \sim Beta(1, A)\ .$$

Corollary

The random walk crosses the diagonal only finitely many times.

Weighted preferential attachment

Generalized model

Probability of assigning new edge to a vertex of degree k is proportional to $\mathcal{W}(k)$ for some *weight function* $\mathcal{W} : \mathbb{N} \to \mathbb{R}_{>0}$.

Linear model

$$\mathcal{W}(k) = k + \beta,\ \ \beta > -1.$$

Convex model

$\mathcal{W}(k)$ is convex and unbounded.

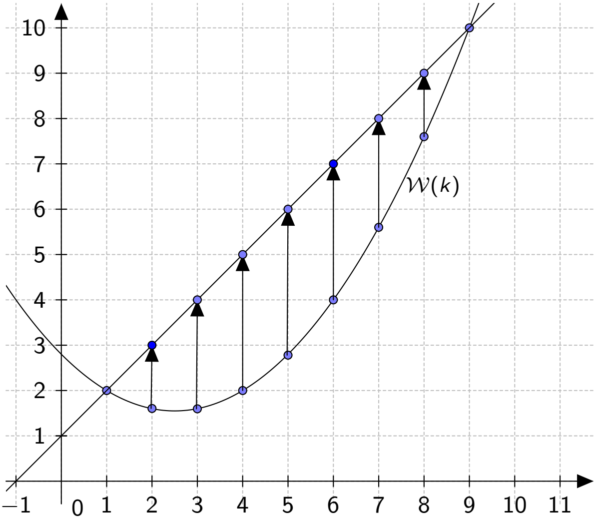
Theorem

The persistent hub appears with probability 1 *also in the linear and convex models.*

Difficulties

Model	Total weight of the vertices is random	All pathes are equally likely	Beta limiting distribution
Basic	No	Yes	Yes
Linear	No	Yes	Yes
Convex	Yes	No	No

Comparison with the linear model



Thank you!