Gravitational Radiation:
From theory to detection

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October, 2013

Summary

Gravitational waves are ripples in space and time which locally stretch and compress space. These waves are strongly emitted from the most violent astrophysical systems such as two black holes orbiting each other. However, the stretching and distorting we find on Earth from these sources is extremely small, of the order of 1/1000 the diameter of a proton! This forms a huge challenge to detect these waves however it is possible using many different methods and we explore these within.

Introduction

The existence of gravitational waves, which are propagating ripples in space-time, are predicted from Einstein's Theory of General Relativity. These ripples are emitted from all accelerated, asymmetrical masses however their magnitude is extremely small when they finally reach us on Earth. For any hope of detecting these wave, we look for the most exotic and violent events in the cosmos. These sources include colliding neutron stars, merging super-massive black holes, black holes capturing stars, white dwarf binaries, supernova explosions and the most violent of them all, the Big bang itself.

To understand gravitational waves, one must first examine where such a prediction arises from. We begin with a brief introduction to the Landau-Lifshitz harmonic formulation of the Einstein field equations. With the equations in this form, gravitational waves almost fall out immediately. At the lowest order, the vacuum equations yield plane wave solutions which can be put into a specific gauge making the waves transverse. We then discuss the case of waves generated from a source and how at the lowest order, we can use simple arguments and dimensional analysis to derive the quadrupole formula for gravitational wave emission. We then conclude with a discussion of the polarisation of gravitational waves which are the main observables used for detection purpose.

With the theory covered, we may ask the question of how these waves may be detected. While there is no direct evidence as yet for their existence, there is strong indirect evidence obtained from observations of the orbital decay of the binary pulsar PSR1913+16. Purpose built detectors are required for direct detections. The earliest of these are known as 'resonant mass' detectors, which rely on a passing gravitational wave to induce resonant oscillations in a heavy mass. Another type are beam-based detectors which make use of Michelson interferometry principles and are currently the most sensitive detectors, with space-based interferometers a proposed project. An astrophysical detection method makes use of an array of millisecond pulsars, which are highly accurate clocks, whose 'ticks' and 'tocks' we measure on Earth will be altered due to a passing gravitational wave. With such a wide variety of methods, each becoming ever more sensitive, a gravitational wave detection is likely imminent.

Theory of gravitational waves

The Relaxed Einstein Field Equations: Straight to waves

The core of the theory of general relativity is encoded into how matter curves space-time and consequently how matter moves in curved spaces. The equations which govern this phenomena are known as the Einstein field equations (EFE) [1]

$$G^{\alpha\beta} = \frac{8\pi G}{c^4} T^{\alpha\beta}. \tag{1}$$

Despite being extremely complicated to solve for all but the simplest systems, the EFE have a clear physical meaning. On the left hand side we have the Einstein tensor $G^{\alpha\beta}$, which is a functional of the metric tensor $g_{\alpha\beta}$, and contains all the information about the curvature of the space-time manifold. On the right hand side, we have $T^{\alpha\beta}$ which is the energy-momentum tensor which represents the matter and energy in the system and also any momenta, pressure and fluxes of such quantities. For the study of gravitational waves, it is much simpler to work with the 'relaxed EFE', which were introduced by Landau and Lifshitz [1]. They are obtained from Eq. (1) via the transformation

$$h^{\alpha\beta} := \eta^{\alpha\beta} - \sqrt{-g}g^{\alpha\beta} \tag{2}$$

and are

$$\Box h^{\alpha\beta} = -\frac{16\pi G}{c^4}\Lambda^{\alpha\beta}, \tag{3}$$

where we must also impose the harmonic gauge condition

$$\partial_\beta h^{\alpha\beta} = 0. \tag{4}$$

The new dynamical variables we solve for are the potentials/wave-field (we will use these terms interchangeably) $h^{\alpha\beta}$ and we note that Eq. (2) is invertible to find back the metric $g_{\alpha\beta}$. The effective-energy momentum tensor $\Lambda^{\alpha\beta}$ is defined as

$$\Lambda^{\alpha\beta} := (-g)(T^{\alpha\beta} + t^{\alpha\beta}_{LL} + t^{\alpha\beta}_{H}). \tag{5}$$

The exact form of the pseudo-tensors $t^{\alpha\beta}_{LL}$ and $t^{\alpha\beta}_{H}$ will not be required here and it suffices to say that they are functions which are quadratic in the potentials. Note also that $\Box = \eta^{\mu\nu}\partial_{\mu\nu}$ is the d'Alembertian wave operator. The set of equations given by Eqs. (3) and (4) solve the EFE of Eq. (1) exactly; no approximations have been made yet. Finally we note that as a consequence of the harmonic gauge condition, we find the conservation equations $\partial_\beta\Lambda^{\alpha\beta} = 0$. In fact, with the explicit expressions for the pseudo tensors, one can show that $\partial_\beta(-g)t^{\alpha\beta}_{H} = 0$ and is hence conserved separately. The conservation equations then reduce to $\partial_\beta(-g)(T^{\alpha\beta} + t^{\alpha\beta}_{LL}) = 0$. These are very important as they can lead us to a definition of the *total* energy for a system including both mass-energy and gravitational energy, which is required when calculating the energy radiated by gravitational waves in this formalism.

[1]The conventions used herein include an event in space-time being labelled by the coordinates $x^\alpha = (x^0, \mathbf{x}^a) = (ct, x^1, x^2, x^3)$, where Greek indices run through all values (0, 1, 2, 3, 4), and the corresponding Latin indices run only through spatial components (i.e. 1, 2, 3). Fundamental constants we use here are c for the speed of light in vacuum and G the gravitational constant. We use $\eta_{\alpha\beta} := \mathrm{diag}(-1, 1, 1, 1) = \eta^{\alpha\beta}$ the Minkowski metric of flat space-time, $g := \det(g_{\alpha\beta})$, $\partial_\alpha := \frac{\partial}{\partial x^\alpha}$ and $g^{\alpha\beta}$ is the contravariant form of the metric such that $g_{\alpha\mu}g^{\alpha\nu} = \delta^\nu_\mu$. We also set the cosmological constant $\Lambda = 0$ since the scales we are considering here are for example binary star systems. A non-zero Λ would have a negligible effect.

Gravitational waves in vacuum

The Landau-Lifshitz harmonic formulation of the EFE puts gravitational waves at ones fingertips. The presence of the the wave operator in Eq. (3) indicates strongly the presence of wave like solutions. The difficulty is the presence of the source term. However if the potentials are weak in the sense that the amplitudes are small ($||\hat{h}^{\alpha\beta}|| \ll 1$), then to a lowest order approximation, $t_{LL}^{\alpha\beta} = 0$ and $t_{H}^{\alpha\beta} = 0$ since they are quadratic in $h^{\alpha\beta}$ and $(-g) = 1$. Under these assumptions the relaxed EFE take the form

$$\Box h^{\alpha\beta} = -\frac{16\pi G}{c^4} T^{\alpha\beta}[\eta], \tag{6}$$

where $T^{\alpha\beta}$ is approximated to lowest order as a function of the Minkowski metric. The next simplification we can make is to examine waves in vacuum, that is where $T^{\alpha\beta} = 0$ so that Eq. (6) now reads

$$\Box h^{\alpha\beta} = 0. \tag{7}$$

The simplest solutions to this equation are the friendly plane waves

$$h^{\alpha\beta} = \mathcal{A}^{\alpha\beta} e^{ik_\rho x^\rho} \tag{8}$$

where $\mathcal{A}^{\alpha\beta}$ are a set of 16 (complex) constants and k_μ is the wave vector. Upon finding a suitable solution we should take the real part of this since this is what has physical significance. The most general solution is of course an arbitrary Fourier sum of plane waves. One can easily show by substituting the plane wave solution of Eq. (8) back into Eq. (7) that this solution exists iff

$$k_\mu k^\mu = 0. \tag{9}$$

This implies that the four-vector $\boldsymbol{k}$ is a null vector so the speed of propagation of this wave is c. Imposing the gauge condition of Eq. (4) gives $\mathcal{A}^{\alpha\beta} k_\beta = 0$ which is a further constraint on the constants $\mathcal{A}^{\alpha\beta}$. It can be shown [2], that one can transform gravitational wave-fields to a transverse, trace-less (TT) gauge. The results of such a transformation to a TT gauge puts further constraints on the wave-field. The transverse condition of the TT gauge implies $\Omega_b \mathcal{A}^{\alpha b} = 0$ where $\boldsymbol{\Omega} := \frac{\mathbf{x}}{r} = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$ is an angular vector which removes longitudinal components of the wave-field. The trace-less condition is more straightforward, it is simply $\delta_{\alpha\beta}\mathcal{A}^{\alpha\beta} = 0$.

We now make a specialisation for a wave travelling in the Cartesian z-direction where $\boldsymbol{k} = (\omega/c, 0, 0, k)$. This implies that $\theta = \phi = 0$ and hence $\boldsymbol{\Omega} = (0, 0, 1)$. From the transverse condition, we find that $\mathcal{A}^{\alpha z} = 0$. From the vanishing of the trace and the previous property we have $\mathcal{A}^{xx} = -\mathcal{A}^{yy}$. The only other remaining terms are $\mathcal{A}^{xy} = \mathcal{A}^{yx}$ where we use the fact that $h^{\alpha\beta}$ is a symmetric tensor (this comes from the symmetry of the metric tensor). Therefore the wavefield becomes, in matrix form,

$$h^{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathcal{A}^{xx} & \mathcal{A}^{xy} & 0 \\ 0 & \mathcal{A}^{xy} & -\mathcal{A}^{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i(kz-\omega t)}. \tag{10}$$

There are only two independent components of the wave. We will see shortly that these are related to polarisation states.

The quadrupole formula for the wave-field

While looking at the situation in vacuum certainly motivates the presence of gravitational waves, in practice we would like to know what the actual amplitudes are for a given system. To do this we need to solve Eq. (6) without setting $T^{\alpha\beta} = 0$. The general solution can be found via a Green's function and is a retarded integral over the past light cone of a chosen field point where we are evaluating the potentials. Our field points of interest would be here on Earth, very far from the sources. In this far-field, the fluctuations are assumed to be weak but are not time independent since the source may have undergone some strong motions to emit these waves and we only receive it much later as a result of retarded times (a simple consequence of the finite speed of light). We also expect only terms in our wave-fields that decay as r^{-1}, where r is the distance to the center of mass of the source, which will dominate over terms of $\mathcal{O}(r^{-2})$ and higher (they are still there; we neglect them due to size).

We seek to find an expansion of this retarded integral solution to the wave-field in terms of a multi-pole expansion in moments. The mass energy density we consider is $\rho(ct, \boldsymbol{r})$. The monopole term $\int \rho \, dV$ will vanish because the total mass-energy is constant, so there are no monopole terms. The dipole term; $\int \rho x^a \, dV$, we also expect to vanish. This is because if one moves to the centre-of-mass frame and orients the origin there, then this moment will vanish due to conservation of momentum. Since the existence of radiation is frame independent, then there is no dipole radiation. Next on the list is the quadrupole moment; $I^{ab} = \int \rho x^a x^b \, dV$ which does not vanish and nor do its derivatives (in general) simply because we have run out of conservation equations to make use of. At the lowest order then, gravitational radiation is quadrupolar in nature.

The quadrupole formula for the wave-field is the bread and butter of gravitational wave physics and describes to lowest order the wave-field of an arbitrary source. This is the link we need to match the amplitude of a wave to the properties of the source that emitted them. Indeed by only the arguments we have made previously and dimensional analysis we can determine this formula up to numerical factors. So far we have found that the potentials scale as (in geometrized units $G = c = 1$ and mass, distance and time have the same effective units) $h^{ab} \sim I^{ab}/r$. The units of I^{ab} are mass times a squared distance so MR^2/r has squared effective units. We need to include time variation somewhere and since the potentials are dimensionless we need a double derivative like $\partial^2/\partial u^2$ which we write as $h^{ab} \sim \ddot{I}^{ab}/r$. Here $u := ct - r = c(t - r/c) =: ct_r$ which is a sort of retarded distance scale related to the retarded time and is included because we know that retardation effects are important when our detector is so far from the source. Restoring SI units, noting that GM/c^2 is a length and GM/c^3 is a time, then we must have

$$h^{ab} \sim \frac{G}{c^4 r} \ddot{I}^{ab}. \tag{11}$$

The constant term $G/c^4 \sim 10^{-44}$ along with the large distances r means that gravitational waves are extremely weak. Detailed calculations based on the retarded integral solution in a far-field approximation show that the constant factor for equality is 2 and we should also convert the moment to the TT gauge.

Polarisation and effect of waves on space time

In general one can expand a gravitational wave field in a linear combination of two basis tensors, say e_{+}^{ab} and $e_{\times}^{ab}$ such that

$$\boldsymbol{h} = h_{+}\boldsymbol{e}_{+} + h_{\times}\boldsymbol{e}_{\times} + \mathcal{O}(r^{-2}). \tag{12}$$

The projection of the wave-field onto each basis tensor then picks out a specific coefficient either h_{+} or $h_{\times}$. For the wave in vacuum travelling in the z-direction from Eq. (10), it is easy to rewrite this in terms of two basis tensors which are orthonormal; the result is

$$h^{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \mathcal{A}^{xx} e^{i(kz-\omega t)} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \mathcal{A}^{xy} e^{i(kz-\omega t)}. \tag{13}$$

By inspection, the independent polarisations for this wave are $h_+ = \mathcal{A}^{xx} e^{i(kz-\omega t)}$ and $h_\times = \mathcal{A}^{xy} e^{i(kz-\omega t)}$. These are reminiscent of the two independent polarisations of electromagnetic waves which are the linear horizontal and vertical states. However, the interpretation is different for gravitational waves. We also stress that an arbitrary weak wave-field can be expanded in terms of these two polarisations, we have just chosen to illustrate the simplest example.

The polarisations are extremely important since they correspond to a key observable for gravitational waves. To illustrate how the polarisations can be observed, we require the metric corresponding to our potentials of Eq. (13). Using Eq. (2) and noting that we are at a lowest order approximation so $(-g) = 1$, then we can easily find $g^{\alpha\beta}$ and invert for the metric $g_{\alpha\beta}$. We choose to display the result in terms of the interval which is defined as $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$, and we find

$$ds^2 = (-c^2 dt^2 + dx^2 + dy^2 + dz^2) + h_+(dx^2 - dy^2) + 2h_\times dxdy. \tag{14}$$

The first piece is the flat-space Minkowski interval and the second piece is due to the small perturbation due to the wave. To see the effects of this interval, let us suppose that we have two freely-falling test masses in the $x - y$ plane (so that $dz = 0$) with the gravitational wave incoming via the z-axis and we consider a slice of time ($dt = 0$). The distance between the masses is then

$$L' = \int ds \tag{15}$$

$$= \int \sqrt{\dot{x}^2 + \dot{y}^2 + h_+(\dot{x}^2 - \dot{y}^2) + 2h_\times \dot{x}\dot{y}}\; d\lambda \tag{16}$$

where a dot indicates differentiation with respect to the parameter λ. Suppose we place the masses at a distance $L = \sqrt{a^2 + b^2}$ and we parametrise the line connecting them by $x(\lambda) = a\lambda$ and $y(\lambda) = b\lambda$. Since the polarisation amplitudes are small, we can make a binomial approximation to find

$$L' \approx L + \frac{a^2 - b^2}{2L} h_+ + \frac{ab}{L} h_\times. \tag{17}$$

This can be written in terms of the relative change in the displacement $\Delta L/L$ where $\Delta L := L' - L$ and we have

$$\frac{\Delta L}{L} = \frac{a^2 - b^2}{2L^2} h_+ + \frac{ab}{L^2} h_\times. \tag{18}$$

Since the amplitudes $h_+, h_\times \propto \cos(kz - \omega t)$, then the actual proper length between the masses oscillates weakly in time. These changes are driven by the polarisations. In the case of the masses aligned along the x-axis, then $b = 0$, $a = L$ and $\Delta L/L = h_+/2$; the same is true for alignment along the y-axis albeit with a minus sign. For alignment at either $+45^\circ$ to the positive x-axis or at -45°, the relative displacement is $\Delta L/L = \pm h_\times/2$. For gravitational wave detection the strain amplitude is defined as

$$h := \Delta L/L \tag{19}$$

and is typically $\sim 10^{-21}$ for astrophysical sources.

What we have just found is that the polarisation h_+ induces changes only along the vertical and horizontal directions but the effects of compression and expansion occur out of phase due

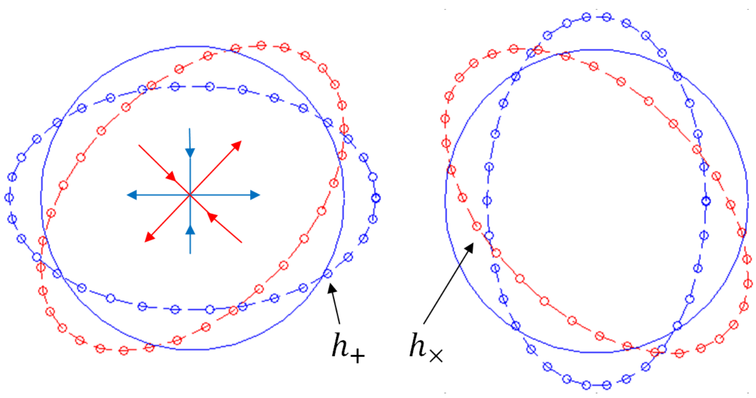

Figure 1: The effect of a passing gravitational wave into the page on test particles at rest lying in a circle. The two modes of oscillation shown are the effects of a purely plus polarised wave and a purely crossed polarised wave. The shape is deformed in such a way that it is compressed in one direction and expanded in another orthogonal to the compression direction. The magnitude of the effects have of course been greatly exaggerated here.

to the minus sign, that is when the horizontal is compressed the vertical expands. The same is true for the $h_\times$ polarisation except its effects are rotated $\pi/4$ relative to those of the h_+. This is the origin of the 'plus' and 'cross' names for the polarisation states and are demonstrated on a ring of test particles in Fig. 1. We stress however that the particles are still at rest relative to the coordinate system; it is the distances that change. Strictly speaking, the distance between infinitesimally close geodesics is time varying. One can think of these deformations as expansions and contractions of the coordinate axes themselves. Arranging many particles in a circle of radius r, one can use Eq. (16) and expand the square root to quadratic order (the integral over the linear terms vanishes) and one would then find the relative change in the radius is $\Delta r/r \propto (h_+^2 + h_\times^2)$. For detection purposes then, it is far better to arrange the masses co-linearly than in more complicated arrangements such as along a circle. Indeed since $\Delta L \propto L$ we should also displace our masses across great distances to amplify the effects.

By observing and measuring these oscillations due to the polarisations, one can hope to detect gravitational waves and this effect forms the basic principle for detection methods. The technical challenge however is detecting strains which typically have amplitudes, for astrophysical sources, of 10^{-22} or one part in 10^{22}. This is far less than even an atomic nucleus and yet there are detectors gathering data today at these sensitivities. The how will be answered in the latter sections.

Energy emission: The quadrupole formula for power

The effect of gravitational waves emission on the source itself is quite striking. Gravitational waves carry energy and the rate of this energy loss (luminosity) is given by the energy loss quadrupole formula

$$\dot{E}_{gw} = \frac{G}{5c^5}\left(I^{ab(3)}I_{ab(3)} - \frac{1}{3}I^{(3)2}\right) \tag{20}$$

where $I := \delta_{ab}I^{ab}$ and the (3) indicates a third derivative with respect to the retarded time $t_r = t - r/c$. We can obtain some simple scaling estimates from this by considering the case of a binary system that is sufficiently far apart that Newtonian gravity is dominant and tidal forces are negligible. Then $I^{(3)} \sim \mu R^2\Omega^3$ where $\mu := m_1m_2/m := m_1m_2/(m_1+m_2)$ is the reduced mass of the system and Ω is the angular frequency of the orbit, and hence

$$\dot{E}_{gw} \sim \frac{G^{7/3}}{5c^5}m^{4/3}\mu^2\Omega^{10/3} \sim \frac{G^{7/3}}{5c^5}m^{4/3}\mu^2\left(\frac{2\pi}{T}\right)^{10/3} \tag{21}$$

where we have involved Kepler's third law $\Omega^2R^3 = Gm$ and the period $T := 2\pi/\Omega$. The pre-factor $G^{7/3}/5c^5 = 1.5\times10^{-67}$, so the energy emitted is typically weak. For instance the

Earth-Sun system, emits (according to Eq. (21)) only 6 Watts (a more careful analysis gives about $6 \times 32 \approx 200$W) which is small compared to the energy the sun radiates via electromagnetic radiation which is $\sim 4 \times 10^{36}$W. However, for extreme systems such as a super-massive black hole binary with masses of the order of $10^6 M_\odot$ and period of one year [3], we find an emission of $\sim 10^{32}$ W. A stronger source of radiation will be when the black holes collide, where their emissions can reach $c^5/G \approx 10^{53}$W.

To obtain higher order solutions to Eq. (3) and hence higher-order quadrupole like formulae one typically employs a weak field, slow motion approximation. The potentials $h^{\alpha\beta}$ and the effective energy-momentum tensor $\Lambda^{\alpha\beta}$ are then expanded in a power series about the small parameter $\epsilon := v_T/c$ where v_T is a characteristic velocity of the system and the resultant equations are matched order-by-order and solved iteratively. This process is known as the post-Newtonian approximation and we refer the reader to the papers of Pati and Will [4, 5], Futamese and Itoh [6,7] and Blanchet and Damour [8,9] where these authors have carried out the calculations to the 3.5PN order! The resultant formulae are important because v_T may become comparable to c such as in a binary merger where the velocities can approach $\sim 0.5c$, therefore even terms with high powers of ϵ become important. These high-order expansions are also used as theoretical templates to sift out the true wave signal from data collected at detectors such as LIGO and VIRGO and are therefore extremely sought after to high orders (currently 3.5-4PN).

Detection methods

Indirect evidence: The binary pulsar PSR1913+16

We have seen that gravitational waves carry energy so it is natural to ask then what conservation of energy has to say about this. We consider a binary system that would satisfy the conditions of the quadrupole formula, that is weak fields and slow motions. The Newtonian orbital energy is $E_{orb} = -Gm_1m_2/(2R) = -Gm\mu/(2R)$ and if we suppose that energy is conserved so that the rate of decrease of the orbital energy is exactly matched by the rate at which energy is transported due to gravitational waves, then $\dot{E}_{orb} = -\dot{E}_{gw}$. Taking derivatives and using Kepler's third law we can find differential equations relating the parameters of the motion assuming a quasi-circular orbit; they are

$$\dot{R} \sim -\frac{G^3}{c^5}\frac{m^2\mu}{R^3}, \quad \dot{\Omega} \sim \frac{G^{5/3}}{c^5} m^{2/3}\mu\,\Omega^{11/3}, \quad \dot{T} \sim -\frac{G^{5/3}}{c^5} m^{2/3}\mu \left(\frac{2\pi}{T}\right)^{5/3}. \tag{22}$$

From these equations we can see that the emission of gravitational waves decreases the separation and the period of the bodies while also increasing the angular frequency (these conclusions are the same for elliptical orbits). The decrease in the separation is difficult to observe due to the large distances to astrophysical sources, however for sufficiently strong systems it should be possible to measure the decay of the period (or angular frequency) over a long time of observations. The question is now finding such a source out in the cosmos.

The hallmark example of these effects of gravitational wave emission on binary orbits was first observed in the binary pulsar PSR1913+16. This system was discovered by Hulse and Taylor in 1974 [10] by detecting radio pulse emissions from an active radio pulsar. They were able to conclude there was also an inert neutron star companion in orbit with a period close to 8 hours. From subsequent observations, the period has seen to decrease at a rate of 76.5 microseconds per year with decreases in the separation as well. If one uses a a form of Eq. (22) for the period with the correct numerical constants along with the known parameters of the orbit [11], then the prediction of the rate of decrease of the period by the quadrupole formula is $\dot{T} = -2.402 \times 10^{-12}$ s/s which is in excellent agreement with the observed value

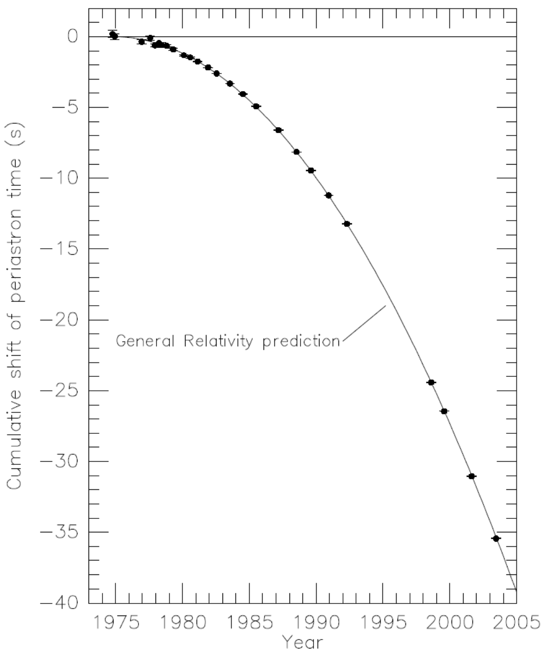

Figure 2: The decay of the orbit for the Hulse-Taylor binary. The solid parabolic line is the theoretical results from General relativity and the data points indicate the change in time it takes for the system to reach perihelion. The solid flat line at the top of the plot represents no orbital decay which is the Newtonian gravitational prediction (reproduced from [11]). There is brilliant agreement between the General relativity prediction and observations.

of $\dot{T} = -(2.427 \pm 0.026) \times 10^{-12}\,\mathrm{s/s}$ [12] This observed period decrease is in remarkable agreement with the GR prediction and is thus seen as an indirect observation of the existence of gravitational waves. The agreement has been continually observed over thirty years and in Fig.2 we show the comparison between the experimental data and the theoretical prediction of the orbital decay. This discovery also won Hulse and Taylor the Nobel prize for physics in 1993.

Resonant mass detectors

Resonant mass detectors or "bar" detectors were historically the first proposed detection method for gravitational waves. In 1959 [13], Joseph Weber proposed the use of a piezoelectric crystal (an induced electric field would aid sensitivity in this material) as a means of observing the strain induced by a passing gravitational wave. The simplest model of this extended system is two masses connected by a string. Vibrations will be induced in the system as the masses seek to move along the time-varying geodesics created by the passing wave. The masses will also experience a linear restoring force and mechanical dissipations as the spring oscillates and this is used to model the electrical forces of attraction of atoms within the crystal. Weber found that the equations of motion took the form of a simple harmonic oscillator with a natural frequency and an oscillatory driving term due to the gravitational wave. The advantage of an extended body is that one can choose to observe only single modes of oscillation due to the relative sizes of the quadrupole moments. The detector will also be left oscillating in a "ringdown" phase after the wave passes which implies a better time range for measurement over two unconnected bodies. The detection procedure was now at least in principle clear: construct a heavy detector (increased quadrupole moment) with a natural frequency close to the frequencies of the sources you wish to detect and the system will resonate.

Weber built his detectors out of massive cylinders of aluminium with piezoelectric crystals used around the cylinders surface to convert the modes into electrical signals. The use of a metal such as aluminium is due to to its high Q value in its lowest mode. The Q value is a measure of how well energy is 'stuck' in the resonator causing oscillations; a high Q material will oscillate for longer. Even with a 3 metre long, 1000 kg cylinders set to detect frequencies of $\sim$ 1.5kHz, a wave with amplitude 10^{-21} will only induce changes of the order of $\Delta L \sim hL \sim 10^{-21}$m. His detectors were set to measure displacements of 10^{-16} m [14] and indeed he claimed many detection events over the years around 1970 [14–17]. Unfortunately detectors built to higher sensitivities with detectable strains of 10^{-17} found null results [18,19] putting Weber's 'detections' under question. Regardless of this, Weber's vision was responsible for beginning the push towards combining gravitational wave physics and experimental methods.

There are numerous sources of noise in bar detectors which makes reaching high sensitivities a difficult task and these must be controlled in order to measure strains of order $10^{-20} - 10^{-21}$. We discuss these below:

- *Thermal noise:* Thermal noise is due primarily to Brownian motion. The root mean square (rms) of thermal vibration amplitudes is [20]

$$\left\langle \delta L_{rms}^2 \right\rangle^{1/2} \propto \frac{1}{f}\sqrt{\frac{T}{M}} \tag{23}$$

  where T is the temperature of the detector, M its mass and f the frequency of the oscillation mode which is close to the gravitational wave frequency for resonance. From this equation we can see that we should do three things to minimize thermal noise: find a material (and a corresponding source) with a large frequency, have a massive material (hence Weber's 1000kg cyclinders) and a low temperature. Indeed modern resonant detectors, such as AURIGA and NAUTILUS in Italy and EXPLORER in Switzerland, which join in detection to decrease chance events [21–23], are operated at ultra-low cryogenic temperatures such as 100 mK. Even at these temperatures, typical rms amplitudes are of the order of 10^{-18}m, which would swamp any gravitational wave signal. The workaround here is the high Q value which is of $\sim 10^6$ [24]. The time it takes for the thermal vibrations to take effect is now $Q/f \sim 1000$s for 1kHz, while the gravitational wave cycles every 1ms. In this 1ms, the random walk amplitude is $Q^{1/2}$ smaller, so now

$$\left\langle \delta L_{rms,1ms}^2 \right\rangle^{1/2} \propto \frac{1}{f}\sqrt{\frac{T}{QM}} \tag{24}$$

  and it is then possible to reduce the thermal noise to $\sim 10^{-20}$m.

- *Electrical and Vibrational:* These are sources of electrical noise due to the quality of the devices used to convert the mechanical vibrations into electrical signals. These are minimized by using the latest SQUIDs (Super-conducting Quantum Interference devices) which work to amplify these low frequencies. Vibrational noise can come from seismic sources or man-made causes such as freight-trains or other heavy vehicles. Storms can also have an effect on the vibrations in the bar. Reducing noise of this type involves suspension of the bar from systems of pendulums and is also a feature seen in ground-based interferometry detectors to hold the masses at either end of the arms [25].

- *Quantum limits:* The extremely small sensitivities required by gravitational wave detectors also opens up the possibility of reaching insurmountable quantum limits. The quantum limit introduces zero-point vibrations in the bar which for a 1kHz wave, gives a quantum rms length of $\sim 10^{-21}$m which is of the order of the strains predicted for astrophysical sources. A simpler source of a quantum limit can be seen from the uncertainty principle

$$\Delta x \Delta p \geq \hbar/2. \tag{25}$$

  A change in length causes an uncertainty in the momentum which is approximately $\Delta p \sim M\Delta v \sim Mf\Delta x$. Therefore

$$\Delta x \gtrsim \left(\frac{\hbar}{2Mf}\right)^{1/2} \sim 3 \times 10^{-20} m \tag{26}$$

for a 10^3 kg bar with 1 kHz oscillations. Therefore the quantum limit implies a minimum strain sensitivity of $h = \Delta x/x \sim 10^{-21} - 10^{-20}$ for a bar with length x of a few metres.

It is therefore difficult to reach the strains of $\sim 10^{-21}$ with resonant detectors.

Other types of resonant detectors include spherical detectors which can reach higher sensitivities than cylindrical versions. The advantages of a spherical detector include a higher energy-cross section than a bar detector simply due to the geometry of the object. Some of these spherical detectors include the miniGRAIL [26] detector in the Netherlands which is a 1300 kg, 68cm diameter Copper-Aluminium alloy sphere with peak sensitivities of 4×10^{-22} operated at temperatures around 40 μK. There is also the Mario Schenberg detector in Brazil which had target sensitivities of 2×10^{-21} $Hz^{-1/2}$ around 3.2 kHz [27] and is still having its systems improved for better noise reduction [28,29]. The last comissioned runs of these detectors were in 2010 and 2008 respectively [30], which is now characteristic of resonant detectors which are being phased out in favour of beam variants which can reach higher sensitivities.

While it may be the end for these large scale resonant detectors, small resonant detectors on the scale of centimetres may be the future of this type of wave detection. Goryachev and Tobar of the University of WA [31] have proposed the use of a 2.5 cm diameter quartz-bulk-acoustic-wave (BAW) cavity that would be cooled to around 20 mK and with the use of highly sensitive SQUIDs could detect strains around 10^{-22}. A passing gravitational wave in the frequency range of 1-1000 MHz would produce resonances in the 2 mm thin disk. The curvature of this BAW then traps the phonons and hence increasing signal-to-noise ratios with very high Q values between $10^6 - 10^{10}$. Theorised sources for such a device to detect include black holes covered in dark matter, plasma flows and possibly cosmic strings. It may remains to be seen whether such sources do exist in this high frequency band. This small and highly sensitive device, if it can be manufactured to be able to reach these sensitivities, is certainly a promising prospect.

Beam detectors

Interferometry

The ground based interferometer is a highly sensitive measurement technique and is therefore well aligned in principle to measure the strains from a gravitational wave over wide frequency bands. The simplest version of such a device involves high-powered laser light, which is coherent, passing through a beam splitter and travelling along two arms. The light then reflects off a mirror connected to a very large mass. The light from the arms is then eventually recombined at a detector. A passing gravitational wave will alter the proper lengths between the masses and the beam splitter in each arm to different extents, which means that the travel time for each arm changes. Recombining the laser light we find a phase difference and hence an interference pattern.

To analyse the interferometer set-up, we consider for convenience a wave travelling into the plane of the detector that is purely plus polarised and the arms of the detector are aligned along the x and y directions. We assume the wavelength of the wave is far greater than that of the arm lengths and that we are of course in the far-field of the source. The passing wave will induce expansions and contractions along each arm (simultaneous expansions in x, contractions in y and vice versa) with a frequency that of the wave. The phase difference when recieved at the detector, assuming they are in phase to begin with, is

$$\Delta\phi = \frac{2\pi\Delta L}{\lambda} = \frac{2\pi L}{\lambda}h, \tag{27}$$

where λ is the wavelength of the laser light. We can see that $\Delta\phi \propto h$ so strains from the wave will induce phase shifts. The actual wave amplitude measured at the detector is a linear combination of the two polarisation states and is written as

$$h(t) = F_+ h_+(t) + F_\times h_\times(t), \tag{28}$$

where F_+ and $F_\times$ are known as antenna patterns, which take into account the angles between the plane of the detector and the source and are of order unity. In practice one instead makes use of the *nulling method* where one modifies the initial phases of the laser light so that without a gravitational wave signal, there is destructive interference at the detector. A wave signal would then introduce an error in this 'dark spot' which can be measured more acutely than an interference pattern because of the small size of h.

There are many different interferometer types used to make the phase shift of Eq. (27) detectable. The first is a *delay line interferometer* where mirrors are placed in each arm so that the light reflects and bounces many times. This increases the effective path length of the arm so for example a 3km arm with 50 reflections has an effective length of 150m which increases L and thus increasing $\Delta\phi$ larger. There are disadvantages with this approach because the reflected light can scatter off other bouncing light which can cause a phase shift in the light that eventually leaves that arm. Combining the two arms may then give a false signal because of the phase obtained due to scattering. This can be partially nullified by modifying the frequency of the light. Another disadvantage is a delay line interferometer requires large mirrors, which in turn requires larger vacuum chambers to contain them. Another type is the *Fabry-Perot cavity interferometer* where two extra highly-reflective mirrors are inserted into each arm. The light is then bounced in these cavities and forms a resonance which leads to very sharp frequencies passing out of the cavity. The effectiveness of these devices depend on very stable laser frequencies since changes in the frequency can alter the very precise geometry required for resonance. The advantages over the delay line type include smaller mirrors and no scattering issues since all the light travels along the same paths. Real interferometers used for detection are built from these two basic types of interferometer.

Just as for resonant detectors, noise is is the challenging factor that must be reduced to obtain positive wave detections. Sources of noise for a beam detector are a little different for a resonant detector because of the difference in the measurement technique: the former use electromagnetic radiation while the latter makes use of modes of oscillations in solids. Some of the sources of noise in interferometers include:

- *Photon Counting (Shot) noise:* The shot noise is found in the signal at the detector. This limit is set due to fluctuations in the number of detected photons and is analogous to how one can measure the rate at which rain falls, but the actual numbers of droplets at any given time is changing. These fluctuations alter the phase of the light waves since the number received $N = N_0 \sin^2(\Delta\phi/2)$ where N_0 is the initial number from the laser. For the gravitational wave detector LIGO, the wavelength of the 10 W laser is $\sim 1\mu m$ [32] with arms of $L \sim$ 3km, so for a typical strain of $h \sim 10^{-21}$, the phase change is only $\Delta\phi \sim 10^{-11}$ rads. Inverting the relationship using a small angle approximation, we can find an uncertainty in the change in length which can be related to a minimum strain we can hope to detect. The shot noise is frequency dependent and is a dominant source of noise for frequncies greater than 200 Hz. This is largely the reason why the noise curves of the ground interferometers in Fig.3 rise sharply around the 10^3 Hz range.
- *Radiation pressure:* This type of noise is not only a limit on the strains we can detect, but also on the lasers we can use. Increasing the laser power will increase the number of photons bouncing off the mirrors and since photons carry momentum we have random

fluctuations of the mass holding the mirror because of an uncertainty in the momentum deposited. A rough estimate on this limit to the strain can be obtained from Eq. (26) where we instead use $\Delta p \sim m\Delta L/\tau$ where τ is an elapsed measuremnt time, to obtain

$$h_{min} \sim \frac{1}{L}\left(\frac{\tau\hbar}{M}\right)^{1/2} \sim 10^{-23} \tag{29}$$

for $M \sim 100$kg, $\tau = 1$ms and $L = 3$km. We could reduce this value practically by increasing the masses holding the mirrors so that their recoil is even less than that of the changes in length we need to detect for a given strain. The quantum limit is similar to the analysis for the radiation pressure and is due to entirely to the uncertainty principles between conjugate variables. A proposed way [33] to reduce these sources of noise is to use squeezed states of light where two types of phase obey a Heisenberg uncertainty principle. The squeezed state has a decreased uncertainty in one state and hence an increase in another. This can be used to decrease the shot noise below the quantum limit, if one only requires information about one type of phase.

- *Vibrational noise:* Vibrational noise is largely due to seismic motions. The key to reducing seismic noise is to operate the detector at frequencies where seismic noise is minimal. This typically occurs at frequencies around 200 kHz and above. It is true that seismic motions will affect the displacements ΔL but these can be minimised by picking the right frequency ranges to detect waves in. A major reason why the noise curves in Fig. 3 for the ground interferometers increase at lower frequencies is due to an increased contribution of seismic noise. The interferometers are still protected against vibrational noise and this is the realm of sophisticated mechanical technologies as discussed by Ju, Blair and Zhao [34]. This involves holding the mirrors with systems of springs and pendulums.

The minimum achievable noise when considering shot noise and radiation pressure occurs when the laser power is well configured: if the power is too high, the radiation-pressure dominates; while if it is too low, shot noise takes over. Other sources of noise are thermal motions which are minimised by supercooling the mirrors and extreme vacuum tubes for the laser light to travel without interference and scattering.

Ground based interferometers are the current most sensitive wave detectors and form a network around the world. The reason for many detectors is that a positive detection at one site may be only a coincidence event. If the same signals are detected by another project elsewhere on the globe at around the same time, then a coincident event may be ruled out in favour of a real detection. It is also necessary to have at least three detectors for triangulation of the source. Detectors in this network include the LIGO (Laser Interferometer Gravitational wave Observatory) project [35] where two interferometers (one 4km and one 2km arm) are situated in Hanford, Washington and another (4km) in Livingston, Louisiana in the US. There is also VIRGO in Cascina, Italy, with 3km arm lengths set to detect to in the frequency band of 10-10,000 Hz, similar to LIGO. These detectors have reached peak sensitivities of $\sim 10^{-22}$ [36] but as yet no detection has been observed over six runs [37–40]. Other worldwide interferometers include GEO600 in Germany and TAMA in Japan. The next-generation of ground-based interferometers are also in the works, with the advanced LIGO (aLIGO) and advanced VIRGO (aVIRGO) [41] which are designed for sensitivities of $\sim 5 \times 10^{-24}$. These sensitivities will be dominated by quantum noise. It is then likely that these detectors will be the most sensitive one can produce with the architecture of interferometry. It is also expected that the first signals will be observed with these detectors due to the observations of gravitational wave transient signals [42] which are strong sources of waves produced in a binary merger, existing for a short time. A proposed third generation (LIGO, VIRGO being first, aLIGO, aVIRGO second) detector called the Einstein

Telescope (ET) [43] is expected to have peak sensitivities of $\sim 3 \times 10^{-25}$ much less than aLIGO and aVIRGO.

Somewhat more ambitious projects include space-based interferometers. These detectors have the advantage of zero vibrational noise bar mechanical due to the detectors themselves (this can be controlled however) and a wider available frequency band than ground-based detectors which means a wider array of possible sources. The Laser Interferometer Space Antenna (eLISA) [44] is one such proposed space-based interferometer. Apart from being in space, LISA will not follow the L-shape of the ground0based interferometers. Instead it will use three spacecraft with mirrors on each and high powered lasers angled at each spacecraft forming a triangle that will have side lengths of 5 million km, dramatically increasing L and orbit 50 million km from Earth. A passing gravitational wave will alter the distances between the craft and be registered as interference between the laser signals. Space-based detectors are still in proof-of-concept phases.

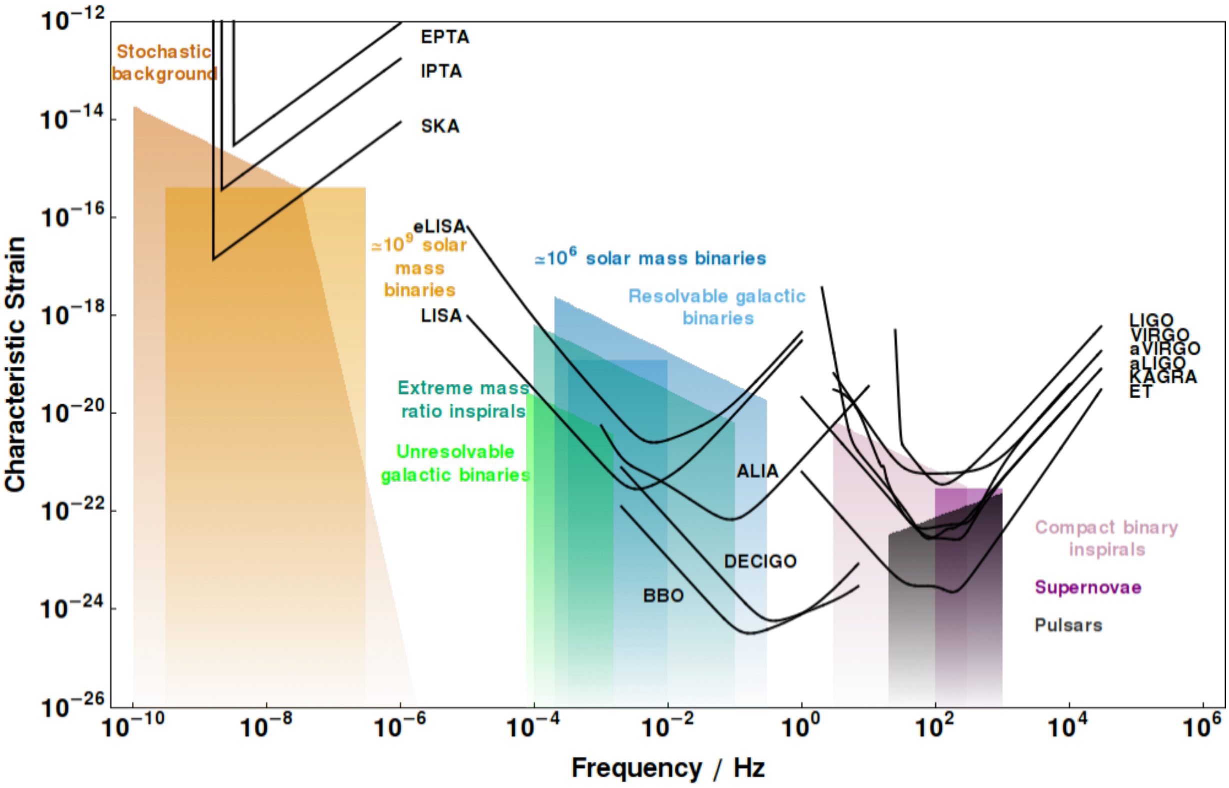

Figure 3: Plots of characteristic strains against frequency ranges. Superimposed are the likely sources in the rectangular blocks and the thin lines are the relevant detectors. The detectors EPTA, IPTA and SKA utilise pulasr timing; eLISA, LISA, ALIA, DECIGO and BBO are space-based interferometers and LIGO, VIRGO, aLIGO, aVIRGO, KAGRA and ET are ground-based interferometers. We can see that each type of detector is tuned to a specific type of source. The minima in the detector plots represent the floor sensitivities of that detector, which are the maximal operating conditions. For a measurement the strain values must be above these noise curves (Reproduced from the work of Moore, Cole and Berry [45] at the url contained within the reference).

Pulsar timing

Pulsars are rapidly spinning neutron stars that periodically emit pulses of light, which if observing angles are favourably aligned, can be detected on Earth. The times-of-arrival (TOA) of these pulses are very precise where a so called millisecond pulsar can have precisions in the

TOA of 30ns [46]. A passing gravitational wave either through the Earth or at the pulsar will alter the pulse frequency observed similar to a gravitational red-shift effect. By comparing these shifted TOAs to known, unperturbed TOAs, one finds time residuals which are a difference in the TOA. While some portion of the residuals can be explained by other effects such as scattering by interstellar dust or in the Earth's atmosphere, a wave signal should leave an additional contribution to these residuals. Making observations of many pulsar TOAs, say a network of 20-50, can help determine the origins of some of the residuals and hence extract the effects solely derived due to gravitational wave interactions. It can take weeks to observe a pulsar, so data sampling will limit the maximum wave frequencies to $\sim 10^{-7}$ Hz. This is why the the noise curves for pulsar arrays in Figure 3 are centred around the $\sim 10^{-8}$ Hz frequencies. These nano Hz frequency ranges mean that the likely sources to effect a pulsar timing array are super massive black hole binaries and primordial gravitational waves from that may be found in the cosmic microwave background as a relic of the inflationary period.

There are three main pulsar timing array projects on three separate continents. The Parkes Pulsar Timing Array (PPTA) [47] utilising the Parkes radio telescope in Australia. No gravitational wave detections have been made as yet with current data sets used to form bounds for the rates of such wave events [48,49]. The timing array in the United States is known as The North American Nanohertz Observatory for Gravitational Waves (NANOGrav) [50] and in Europe there is The European Pulsar Timing Array (EPTA) [51] which both utilise many observatories for pulsar TOA measurements. Together, these three projects collaborate as the International Pulsar Timing Array (IPTA). To date, no detections have been made. The next generation of pulsar timing detectors is set to be the Square Kilometre Array (SKA) [52] which is to be made of thousands of radio dishes placed in South Africa and in Australia reaching strain sensitivities of $\sim 10^{-15}$ [53].

Conclusion

The existence of gravitational radiation is a firm prediction with their detection a very important test of General relativity. These propagating ripples in space-time as received at the Earth are extremely weak making a chance of detection a monumental challenge. The most sensitive are the beam detectors utilising interferometry along with pulsar timing arrays for the detection of primordial radiation. While no positive detections have been made thus far, it is likely that wave signals will be observed with the next generation of detectors such as aLIGO, aVIRGO and the SKA. Such a confirmation would greatly enhance our ability to probe the universe by introducing an entirely new spectrum of wave signals that scatter very little unlike electromagentic radiation. While it is likely that great leaps forward will be made in understanding currently known astrophysical objects such as black holes and neutron stars, the most exciting discoveries are the unknown unknowns; those objects we do not even know exist.

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