**AISTATS 2020**

# AsyncQVI: Asynchronous Parallel Q-value Iteration for Markov Decision Processes

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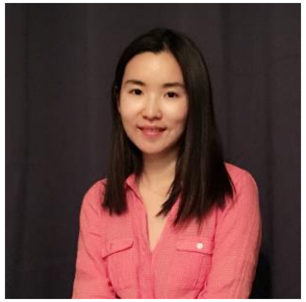
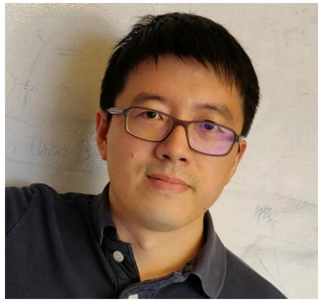


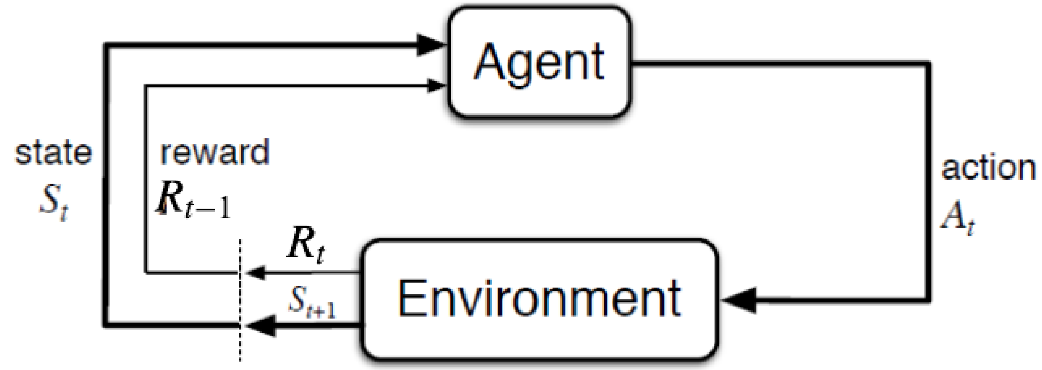
UCLA, Math

Wotao Yin



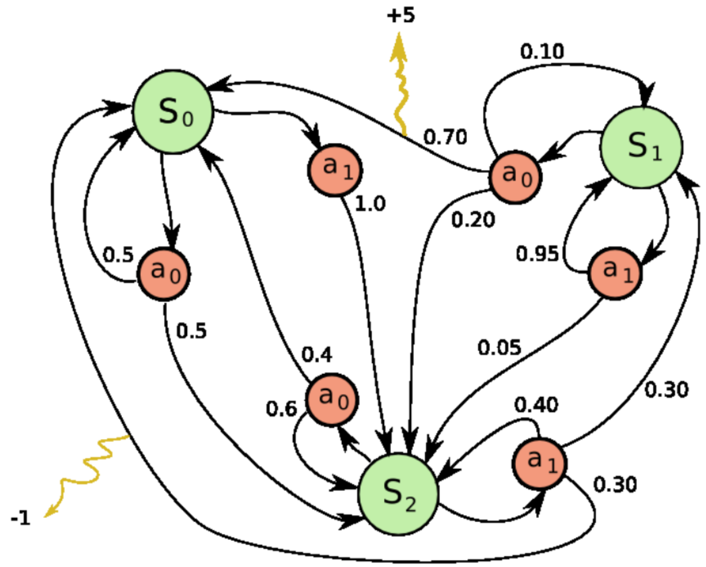
UCLA, Math

# Markov Decision Process




A framework for **Reinforcement Learning.**

- $M := (S, A, p, r, \gamma)$.
- Transition kernel $p(s_{t+1}|s_t, a_t)$ & reward function $r(s_t, a_t, s_{t+1})$

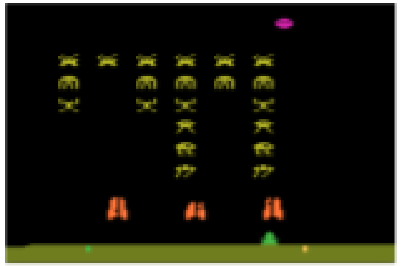



- A **policy** $\pi$: $S \to A$
- We execute $\pi$ to obtain a trajectory: $s_0, a_0, r_0, s_1, a_1, r_1$ …
- **State-value function:**

$$V^{\pi}(s) = E\left[\sum_{t=0}^{\infty} \gamma^t r_t | \, s_0 = s, \pi\right]$$

- **Goal**: find a policy that maximizes the value function $V^{\pi}$.

# Recent Successes and Time Demands in RL



**[DeepMind 2015]**

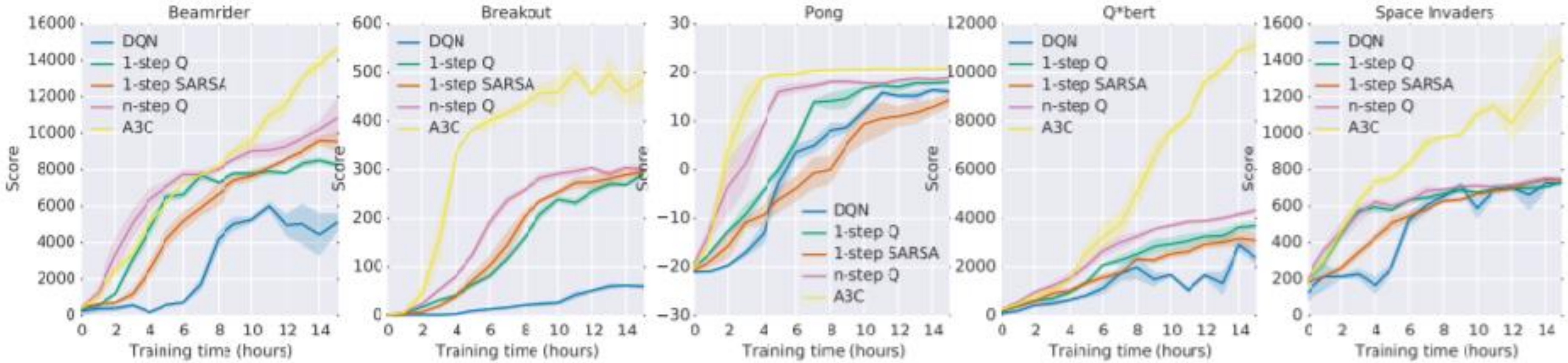
Human-level performance on **Atari 2600**. Trained for **38 days** of game experience

**[DeepMind 2017]**

**AlphaGo Zero.** Trained for **40 days** to surpass all old versions.

**[OpenAI 2019]**

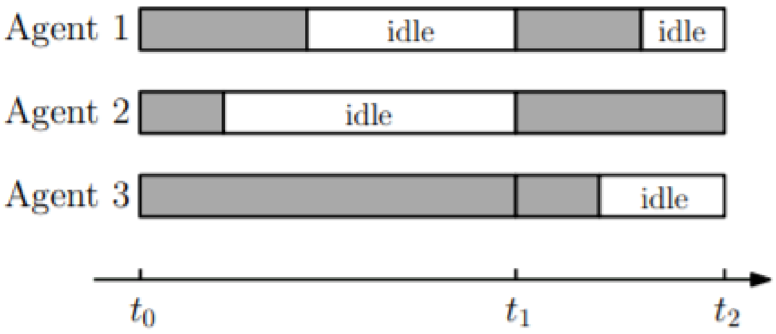
Defeating **Dota 2** world champion. Trained for **180 days**.



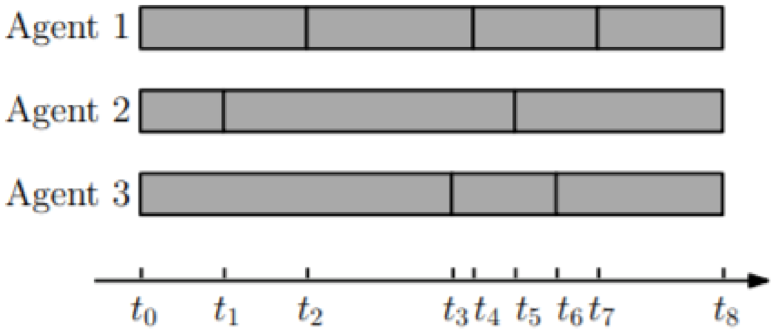

[Minh et al. 2016]

Parallel computing accelerates training.

# Parallel Computing



(a) Sync-parallel computing



(b) Async-parallel computing

**Sync-parallel**

- probably long idle time;
- little tolerant to communication glitches;
- keeps information consistent.

**Async-parallel**

- saves idle time;
- more tolerant to communication glitches;
- easier to incorporate new agents;
- information is delayed or inconsistent.

**Our goal**: a theoretically justified async-parallel algorithm for MDPs.

# Review the Bellman Operator

- Initialize a table $Q_0$ of size $S \times A$.
- Bellman Operator (Q-value iteration):

$$Q_{t+1}(s,a) = T^B(Q_t) = E_{s' \sim p(\cdot|s,a)}[r(s,a) + \gamma \, max_{a'} \, Q_t(s',a') \,] \tag{1}$$

- The fixed point $Q^*$ can induce an optimal policy [Sutton & Barto, 1999].

$T^B$ is friendly for parallel design:

- A nice structure: linear (expectation) + simple nonlinear (max).
- A nice convergence property: $\gamma$-contraction under $|| \cdot ||_\infty$.

# How we build AsyncQVI.

❶ **The original Q-value iteration:**

$$Q_{s,a}(t+1) = \sum_{s'} p^a_{ss'}\left(r^a_{ss'} + \gamma \max_{a'} Q_{s',a'}(t)\right), \quad \forall\ (s,a) \in \mathcal{S} \times \mathcal{A}.$$

Update **ALL** entries per iteration.
Time complexity per iteration: $O(S^2A)$.

❷ **Revise to a coordinate-update fashion:**

$$Q_{s,a}(t+1) = \begin{cases} \sum_{s'} p^a_{ss'}\left(r^a_{ss'} + \gamma \max_{a'} Q_{s',a'}(t)\right), & (s,a) = (s_{t+1}, a_{t+1}); \\ Q_{s,a}(t), & (s,a) \neq (s_{t+1}, a_{t+1}). \end{cases}$$

❸ **Implement in an asynchronous parallel manner:**

$$Q_{s,a}(t+1) = \begin{cases} \sum_{s'} p^a_{ss'}\left(r^a_{ss'} + \gamma \max_{a'} \hat{Q}_{s',a'}\right), & (s,a) = (s_{t+1}, a_{t+1}); \\ Q_{s,a}(t), & (s,a) \neq (s_{t+1}, a_{t+1}). \end{cases}$$

❹ **Approximate with Samples:**

$$Q_{s,a}(t+1) = \begin{cases} \frac{1}{K} \sum_{k=1}^{K}\left(r_k + \gamma \max_{a'} \hat{Q}_{s'_k,a'}\right) - c, & (s,a) = (s_{t+1}, a_{t+1}); \\ Q_{s,a}(t), & (s,a) \neq (s_{t+1}, a_{t+1}) \end{cases}$$

# How we build AsyncQVI.

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Update **ONE** entry per iteration.
Time complexity per iteration: $O(S)$.

3. **Implement in an asynchronous parallel manner:**

$$Q_{s,a}(t+1) = \begin{cases} \sum_{s'} p_{ss'}^{a}(r_{ss'}^{a} + \gamma \max_{a'} \hat{Q}_{s',a'}), & (s,a) = (s_{t+1}, a_{t+1}); \\ Q_{s,a}(t), & (s,a) \neq (s_{t+1}, a_{t+1}). \end{cases}$$

4. **Approximate with Samples:**

$$Q_{s,a}(t+1) = \begin{cases} \frac{1}{K} \sum_{k=1}^{K}(r_k + \gamma \max_{a'} \hat{Q}_{s_k',a'}) - c, & (s,a) = (s_{t+1}, a_{t+1}); \\ Q_{s,a}(t), & (s,a) \neq (s_{t+1}, a_{t+1}) \end{cases}$$

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Parallel run with **N** agents.
Scatter computing load.
Asynchronous causes information delay.

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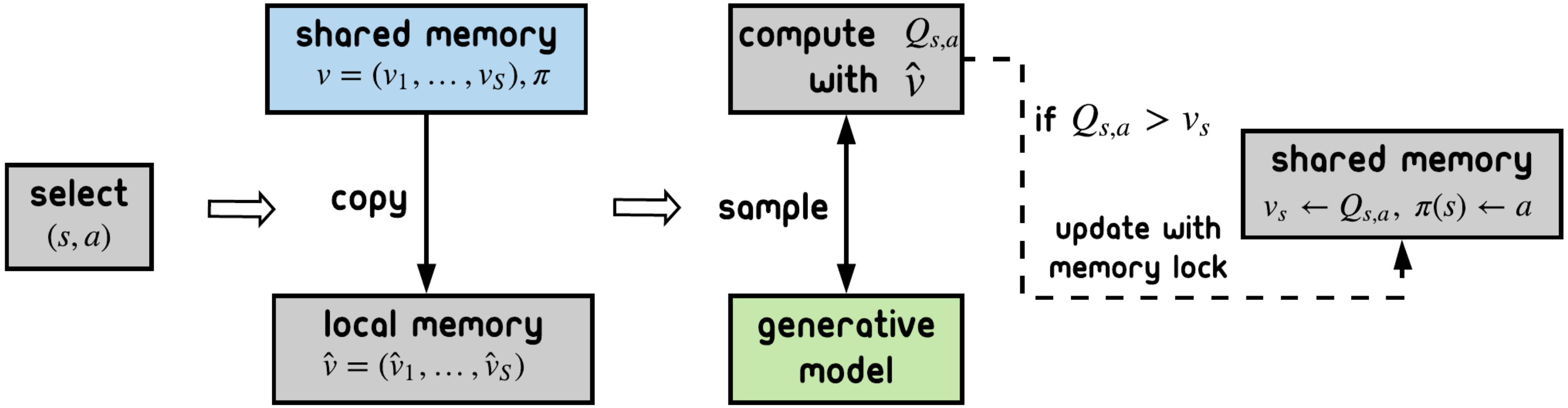
**Generative Model** (GM):
Input: any $(s, a)$
Output: $(s', r) \sim p(\cdot\,|s, a)$

Stochastic approximation with a generative model.
Scatter sampling load.

# AsyncQVI

N computing agent continuously and asynchronously do:




- Selection can be random or cyclic.
- Memory complexity: $O(S)$;
- The copying can be done in a less frequent fashion;
- The overhead of updating lock is neglectible;
- Parallel speedup: N times faster (ideally).

# Theoretical Guarantee

## Assumptions [partial asynchronism]:

1. Delay is uniformly bounded by $B_1$;
2. The time interval between consecutive updates for each entry is bounded by $B_2$.

**Theorem 1 (Zeng, Feng, and Yin 2020)**

*Under the above assumptions, with probability at least $1-\delta$, AsyncQVI returns an $\varepsilon$-optimal policy using samples*

$$\tilde{\mathcal{O}}\Big(\frac{B_1+B_2}{(1-\gamma)^5\varepsilon^2}\log\Big(\frac{1}{\delta}\Big)\Big).$$

If $B_1 + B_2 = O(SA)$, our sample complexity is near-optimal.

The sample complexity of single-thread methods with a generative model is:

$$\Theta\Big(\frac{SA}{(1-\gamma)^3\varepsilon^2}\log\Big(\frac{1}{\delta}\Big)\Big)$$

[Azar et al. 2013, Agarwal et al. 2019]

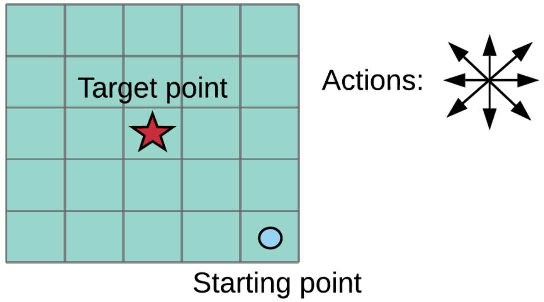
# Related Works

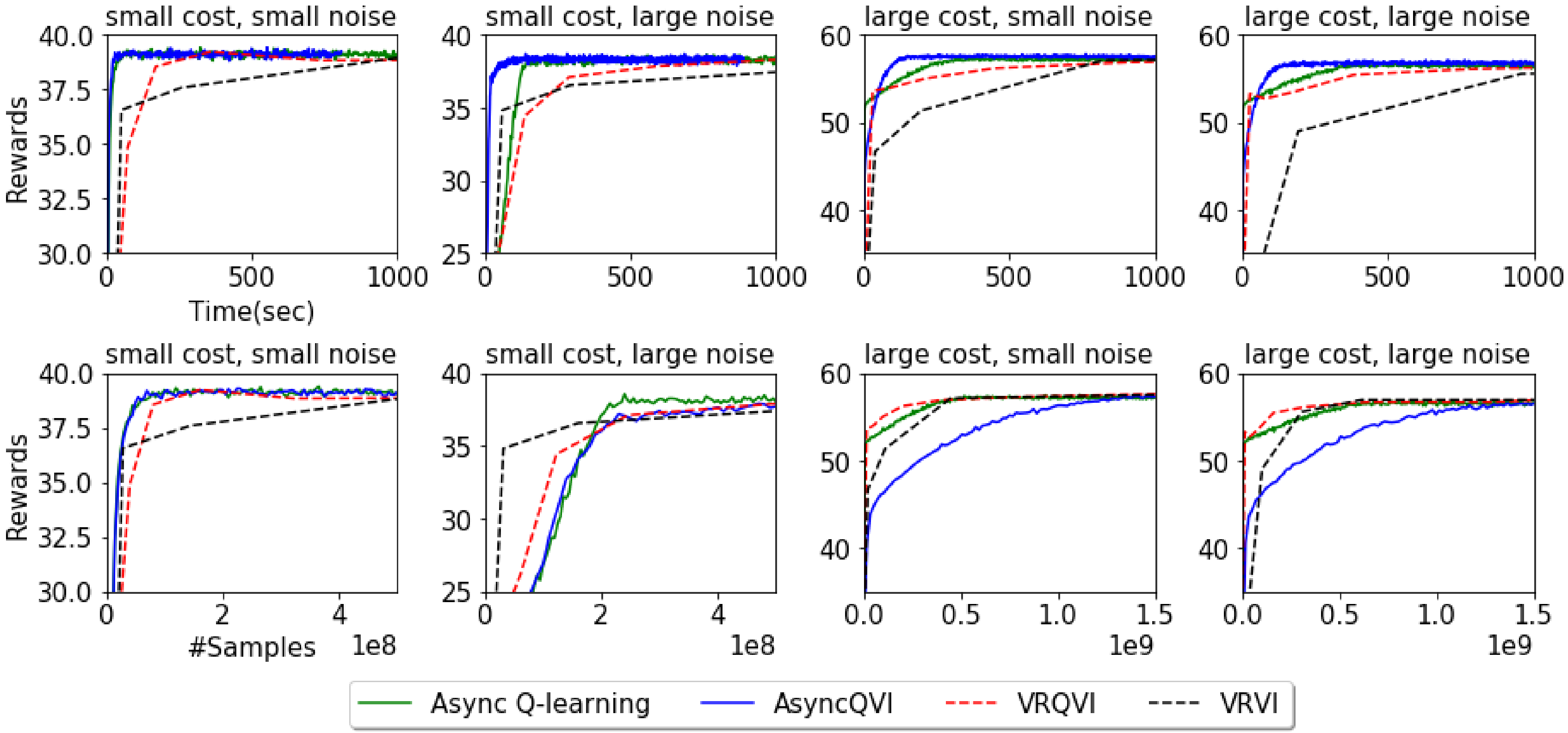
| Algorithm | Setting | Async-parallel | Sample | Memory |
|---|---|---|---|---|
| Totally Async QVI[2] | Full knowledge | Unbdd delay | N/A | $\mathcal{O}(SA)$ |
| Partially Async QVI[3] | Full knowledge | Bdd delay | N/A | $\mathcal{O}(SA)$ |
| Async Q-learning[4] | RL | Unbdd delay | N/A | $\mathcal{O}(SA)$ |
| VRVI[5] | Generative model | $\times$ | $\tilde{O}(\frac{SA}{(1-\gamma)^4\varepsilon^2}\log(\frac{1}{\delta}))$ | $\mathcal{O}(SA)$ |
| VRQVI[6] | Generative model | $\times$ | $\tilde{O}(\frac{SA}{(1-\gamma)^3\varepsilon^2}\log(\frac{1}{\delta}))$ | $\mathcal{O}(SA)$ |
| AsyncQVI | Generative model | Bdd delay | $\tilde{O}(\frac{SA}{(1-\gamma)^5\varepsilon^2}\log(\frac{1}{\delta}))$ | $\mathcal{O}(S)$ |

AsyncQVI trades a few more samples for less running time and memory.

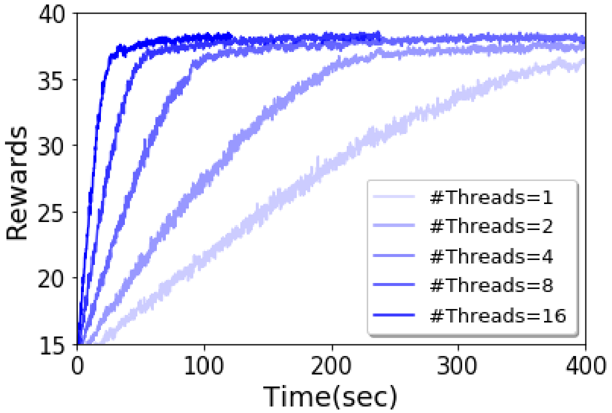
[2] Bertsekas and John N Tsitsiklis 1989.
[3] Bertsekas and John N Tsitsiklis 1989.
[4] John N. Tsitsiklis 1994.
[5] Sidford, Wang, Wu, and Ye 2018.
[6] Sidford, Wang, Wu, L. Yang, et al. 2018.

# Numerical Experiment




- 100*100 grid world;
- Noises in transition;
- A big reward at the target;
- Minor traveling costs.




Parallel algorithms are run with 20 threads.

# Conclusion:

- We propose an async-parallel algorithm for MDPs with a near-optimal sample complexity.
- AsyncQVI trades a little more samples for less time and memory.
- AsyncQVI has linear parallel speedup empirically.

# Future work:

- Involve exploration.
- Consider function approximation.

*Thank you!*