Problem 1: Solve the following recurrence relations:

(a) \( a_n = 3a_{n-1} + 4a_{n-2} - 12a_{n-3} + n^2 \) for \( n \geq 3 \) and \( a_0 = 1, a_1 = 2, a_2 = 3 \).

(b) \( b_n = b_{n-1} + n \) for \( n \geq 1 \) and \( b_0 = 0 \).

(c) \( c_n = 10c_{n-1} - 21c_{n-2} \) for \( n \geq 2 \) and \( b_0 = -1, b_1 = 1 \).

Problem 2: Suppose we have the recurrence relation \( x_n = c_1x_{n-1} + c_2x_{n-2} \) for \( c_1, c_2 \in \mathbb{R} \). We saw in class that if \( r_1 \) and \( r_2 \) are the roots of the polynomial \( t^2 - c_1 t - c_2 \) and \( r_1 \neq r_2 \), then \( x_n = br_1^n + dr_2^n \) for some \( b \) and \( d \) determined by initial conditions.

(a) Suppose that \( \max(|r_1|, |r_2|) > 1 \), what can you deduce about the long term behavior of \( x_n \)?

(b) Suppose that \( \max(|r_1|, |r_2|) < 1 \), what can you deduce about the long term behavior of \( x_n \)?

(c) Suppose that \( r_1 = 1 \) and \( |r_2| \leq 1 \), what can you deduce about the long term behavior of \( x_n \)?

Problem 3: From Johnsonbaugh:

Chapter 7.2: 14, 16, 27, 37, 48

Problem 4: Solve the following recurrence relation:

\[ a_n = a_{n-1}a_{n-2} \]

with initial conditions \( a_0 = e, a_1 = e^2 \), where \( e \) is the base of the natural logarithm.