Math 33 B Second Hour Exam (Practice)

1. Find the power series expansion for $\frac{e^x}{1+x}$ out to the x^4 term

2. Find the power series expansion of $\tan^{-1} x$, about 0.

3. Find the power series expansion for e^x about $x=1$ and use the remainder term to show it converges to e^x.

4a) Show that if $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = r < 1$, then $\sum_{n=0}^{\infty} a_n$ converges.

b) Show that if $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = r < 1$, then $\sum n a_n x^{n-1}$ converges for $|x| < \frac{1}{r}$.

5. Give examples

a) a sequence $f_n(x)$ on $[0,1]$ such that each f_n is continuous and $f_n(x) \to f(x)$ where f is not continuous

b) a sequence $f_n(x)$ on $[0,5]$ such that each f_n is continuous and $f_n(x) \to f(x)$ but $\int_a^b f_n(x)\,dx$ does not converge to $\int_a^b f(x)\,dx$

c) In each case directly show that f_n does not converge uniformly to f (i.e., $\|f_n - f\|_\infty \not\to 0$).