Math 33a Practice first hour exam. **This is a closed book exam.**

1(30). Find the following limits (include calculations!):

a) $\lim_{x\to 0} \frac{x^2}{x+\sqrt{x}}$

b) $\lim_{x\to 0+} x \ln x$

c) $\lim_{x\to 0} \frac{1}{\sin x} - \frac{1}{x}$

e) $\lim_{n\to\infty} \frac{n!}{2^n}$

f) $\lim_{x\to 0} (1+x)^{\frac{1}{x}}$

2(25). Determine which of the following sums converge. Carefully explain which test you are using.

a) $\sum_{n=1}^{\infty} \frac{1}{n^2-n+1}$

b) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^3+k}}$

c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

d) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

e) Determine where the series $\sum \frac{(x+2)^n}{n}$ converges (include a careful picture of the points x for which it converges).

3a) **Define** $\lim_{n\to\infty} x_n = L$.

b) **Prove** that if a sequence x_n converges, then there exists a constant M such that $x_n \leq M$ for all n.

c) **Define**: the series $\sum_{n=1}^{\infty} a_n$ converges.

d) **Prove** that if $0 \leq a_n \leq b_n$ and $\sum b_n$ converges, then $\sum a_n$ converges.