Math 3 J 6/12 Assignment 1

p.96 1. $\lim_{x\to -2} \frac{2x^2+5x+2}{x^2-4}$ $\left[\cancel{= \frac{2(-2)^2+5(-2)+2}{(-2)^2-4} = \frac{2\cdot 4-10+2}{4-4}} = \frac{0}{0} \right]$

$= \lim_{x\to -2} \frac{4x+5}{2x} = \frac{4(-2)+5}{2(-2)} = \frac{-3}{-4} = \frac{3}{4}$

8. $\lim_{x\to \infty} \frac{x^4-2x^2-1}{2x^3-3x^2+3} = \boxed{\cancel{\frac{\infty}{\infty}}}$

$= \lim_{x\to \infty} \frac{4x^3-4x}{8x^2-6x} = \boxed{\cancel{\frac{\infty}{\infty}}}$

$= \lim_{x\to \infty} \frac{12x^2-1}{16x-6} = \boxed{\cancel{\frac{\infty}{\infty}}} = \lim_{x\to \infty} \frac{24x}{16} = \frac{\infty}{16}$
$= \infty$

OR $= \lim_{x\to \infty} \frac{x - \frac{2}{x} - \frac{1}{x^3}}{2 - \frac{3}{x} + \frac{3}{x^2}} = \frac{\infty}{2} = \infty$ [DIVIDE NUMER. & DENOM. BY x^3]

10. $\lim_{x\to 0} \frac{\sin 7x}{x} = \boxed{\cancel{\frac{0}{0}}}$

$= \lim_{x\to 0} \frac{7\cos 7x}{1} = \frac{7\cos 0}{1} = 7$

12 $\lim_{x\to 0} \frac{e^{3x}-1}{1-\cos x} = \left[\cancel{\frac{e^0-1}{1-\cos 0} = \frac{1-1}{1-1}} = \frac{0}{0} \right]$

$= \lim_{x\to 0} \frac{3e^{3x}}{\sin x} = \frac{3e^0}{\sin 0} = \frac{3}{0} = \infty$

Note $\log = \ln$

$(\lambda > 0)$

14. $\lim_{x\to \infty} \frac{\log x}{x^\lambda} = \left[\cancel{\frac{\log \infty}{\infty^\lambda} = \frac{\infty}{\infty}} \right]$

$= \lim_{x\to \infty} \frac{\frac{1}{x}}{\lambda x^{\lambda-1}} = \lim_{x\to \infty} \frac{1}{\lambda x^\lambda} = \frac{1}{\infty} = 0$

16. $\lim_{x\to \infty} \frac{3^x-2^x}{x} = \lim_{x\to \infty} \frac{\frac{3^x}{2^x}-1}{\frac{x}{2^x}} = \lim_{x\to \infty} \frac{\left(\frac{3}{2}\right)^x-1}{\frac{x}{2^x}} = \frac{\infty}{0} = \infty$

26 $\lim_{x\to \frac{\pi}{2}} \frac{\sin x}{x} = \frac{\sin \frac{\pi}{2}}{\left(\frac{\pi}{2}\right)} = \frac{1}{\left(\frac{\pi}{2}\right)} = \frac{2}{\pi}$

43. $\lim_{x \to 0+} x \, e$ ... ? Let $y = \frac{1}{x}$.

a)

$$= \lim_{y \to \infty} \frac{1}{y^3} e^y \overset{\frac{\infty}{\infty}}{=} \lim_{y \to \infty} \frac{e^y}{3y^2} \overset{\frac{\infty}{\infty}}{=} \lim_{y \to \infty} \frac{e^y}{6y} = \lim_{y \to \infty} \frac{e^y}{6}$$

$$= \frac{\infty}{6} = \infty$$

b) similar

c) $x \to 0-$ implies $y \to -\infty$

$$\lim_{x \to 0-} x^n e^{\frac{1}{x}} = \lim_{y \to -\infty} \frac{e^y}{y^n} = \frac{e^{-\infty}}{\infty^n} = \frac{0}{\infty} = 0$$

p.10 7: $3.\underbrace{6143}\underbrace{6143}\ldots$

$$= 3 + \frac{6143}{10\,000} + \frac{6143}{10^8} + \frac{6143}{10^{12}} + \cdots$$

$$= 3 + \frac{\frac{6143}{1000}}{1 - \frac{1}{10^4}} = \text{etc.}$$

OK $\frac{3614}{1000} + \frac{3614}{10^7} + \frac{3614}{10^{11}} + \cdots = \frac{\frac{3614}{10^3}}{1 - \frac{1}{10^4}}$

[use $\underbrace{3.614}\underbrace{3614}\underbrace{3614}\ldots$]

9. $S = \frac{a}{1-r} = \frac{3}{1+\frac{1}{3}} = \frac{9}{10}$