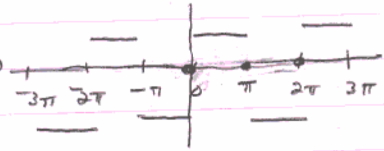
Assignment 8

1. $f(x) = \begin{cases} \pi/4 & 0<x<\pi \\ -\pi/4 & -\pi<x<0 \end{cases}$

$f_o(x)$



$f(-x) = -f(x)$ $\quad$ f is $\underline{\underline{\text{odd}}}$

$\Rightarrow a_n = 0$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_0^{\pi}\frac{\pi}{4}\sin nx\,dx + \frac{1}{\pi}\int_{-\pi}^{0}\left(-\frac{\pi}{4}\right)\sin nx\,dx$$

$$= \frac{1}{4}\left[\frac{-\cos nx}{n}\right]_0^{\pi} - \frac{1}{4}\left[-\frac{\cos nx}{n}\right]_{-\pi}^{0}$$

$$= \frac{1}{4n}\Big(\big(-\underbrace{\cos n\pi}_{(-1)^n} + \underbrace{\cos n0}_{1}\big) + \big(\cos n0 - \cos n(-\pi)\big)\Big)$$

$$= \frac{1}{2n}\left(1-(-1)^n\right) = \begin{cases} 0 & n \text{ even} \\ \frac{1}{n} & n \text{ odd} \end{cases}$$

$$f(x) = \sum_1^{\infty} b_n \sin nx = \sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots$$

$\Rightarrow$ $\underline{\text{NOTE}}$: The book's answer is $\underline{\underline{\text{incorrect}}}$

2. $f(x) = \begin{cases} 0 & -\pi<x<0 \\ 1 & 0<x<1 \end{cases}$

$$(n\neq 0)\quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_0^{\pi}\cos nx\,dx = \frac{1}{\pi}\left[\frac{\sin nx}{n}\right]_0^{\pi} = 0$$

since $\sin n\pi = 0$ for all n

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{\pi}dx = 1$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_0^{\pi}\sin nx\,dx = \frac{1}{\pi}\left[-\frac{\cos nx}{n}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left(-\frac{(-1)^n}{n} + \frac{1}{n}\right) = \frac{1}{n\pi}\left(1-(-1)^n\right) = \begin{cases} 0 & n \text{ even} \\ \frac{2}{n\pi} & n \text{ odd} \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx + \sum_{n=1}^{\infty} b_n \sin nx = \frac{1}{2} + \frac{2}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \cdots\right)$$

15. $f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$

$(n \neq 0)$ $a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} (-\pi)\cos nx\,dx + \int_0^\pi x\cos nx\,dx\right]$

$$= \frac{1}{\pi}\left[\underbrace{(-\pi)\left[\frac{\sin nx}{n}\right]_{-\pi}^{0}}_{=0} + \underbrace{\left[\frac{x\sin nx}{n}\right]_0^\pi}_{=0} - \int_0^\pi \frac{\sin nx}{n}\,dx\right]$$

(since $\sin n\pi = 0$ all n)

$$= \frac{1}{\pi}\left[\frac{\cos nx}{n^2}\right]_0^\pi = \frac{1}{\pi n^2}\left[(-1)^n - 1\right]$$

$$= \begin{cases} 0 & n \text{ even} \\ \frac{-2}{\pi n^2} & n \text{ odd} \end{cases} \quad \text{i.e. } \boxed{\begin{array}{l} a_{2k} = 0 \\ a_{2k-1} = -\frac{2}{\pi(2k-1)^2} \quad k = 1, 2, \ldots \end{array}}$$

$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} (-\pi)\sin nx\,dx + \int_0^\pi x\sin nx\,dx\right]$

$$= \frac{1}{\pi}\left[(-\pi)\left[-\frac{\cos nx}{n}\right]_{-\pi}^{0} + \left[x\left(\frac{-\cos nx}{n}\right)\right]_0^\pi - \int_0^\pi -\frac{\cos nx}{n}\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}(\cos 0 - \cos n(-\pi)) + \left(\pi\frac{(-\cos n\pi)}{n} - 0\right) + \left[\frac{\sin nx}{n^2}\right]_0^\pi\right]$$

$$= \frac{(1-(-1)^n)}{n} - \frac{(-1)^n}{n} + 0 = \frac{1-2(-1)^n}{n} = \begin{cases} -\frac{1}{n} & n \text{ even} \\ \frac{3}{n} & n \text{ odd} \end{cases}$$

i.e., $\boxed{a_{2k} = -\frac{1}{2k},\ a_{2k-1} = \frac{3}{2k-1}}$

$\boxed{a_0} = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0}(-\pi)\,dx + \frac{1}{\pi}\int_0^\pi x\,dx$

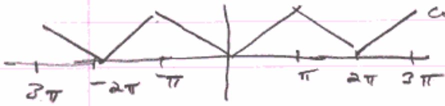
$$= -\pi + \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^\pi = -\pi + \frac{\pi}{2} = \boxed{-\frac{\pi}{2}}$$

$f(x) = \frac{a_0}{2} + \sum_1^\infty a_n \cos nx + \sum_1^\infty b_n \sin nx$

$$= \frac{a_0}{2} + \sum_1^\infty a_{2k-1}\cos(2k-1)x + \sum_1^\infty b_{2k}\sin(2k)x + \sum_1^\infty b_{2k-1}\sin(2k-1)x$$

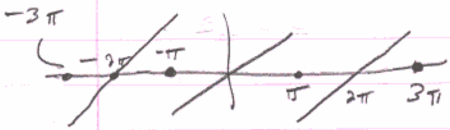
p.370 5. $f(x) = x \quad 0 \le x \le \pi$

cosine expansion: use $f_{ev}(x) = |x|$



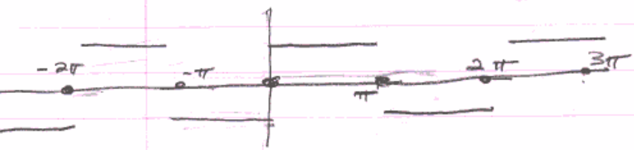
We did this one in lecture!

13. $f(x) = x \quad 0 < x < \pi$ use $f_{odd}(x) = x$



We did this one in lecture!

p.374:1 $f(x) = \begin{cases} 1 & 0 < x < \pi \\ -1 & \pi < x < 2\pi \end{cases}$



Use book's formulas. Note this is just $\frac{4}{\pi} \times$ (first problem in assignment) so you can check it.