Assignment 5

1. p. 149: 2

$$(1-x)^{\frac{1}{2}} = 1 + (\tfrac{1}{2})(-x) + \frac{(\frac{1}{2})(\frac{1}{2}-1)}{2!}(-x)^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-x)^3 + \ldots$$

$$= 1 - \tfrac{1}{2}x + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}x^2 - \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{6}x^3 + \ldots$$

$$= 1 - \tfrac{1}{2}x + \tfrac{1}{8}x^2 - \tfrac{1}{16}x^3 - \ldots$$

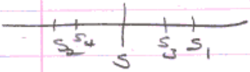
8. $$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - $$

$$\sin x^2 = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \ldots$$

$$\int \sin x^2 \, dx = \frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!} - \frac{x^{15}}{15 \cdot 7!} + \ldots$$

$$\int_0^1 \sin x^2 \, dx = \frac{1}{3} - \frac{1}{7 \cdot 3!} + \frac{1}{11 \cdot 5!} - \ldots = \Sigma (-1)^n a_n$$

It is evident that $a_1 \geq a_2 \geq \ldots$ and $a_n \to 0$ so from proof of alternating series test we have the picture
and $|S - S_n| \leq |S_{n+1} - S_n| = a_{n+1}$



The desired approximation is S_n where $|a_{n+1}| \leq .0001$. We have

$$a_2 = \frac{1}{7 \cdot 3!} = \frac{1}{42} > 10^{-5}$$

$$a_3 = \frac{1}{11 \cdot 5!} = \frac{1}{11 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} > 7.6 \times 10^{-4} > 10^{-5}$$

$$a_4 = \frac{1}{15 \cdot 7!} > 1.3 \times 10^{-5} > 10^{-5}$$

$$a_5 = \frac{1}{19 \cdot 9!} < 1.5 \times 10^{-7}$$

Hence $\underbrace{\int_0^1 \sin x^2 \, dx}_{.3102683017} \approx \underbrace{\frac{1}{3} - \frac{1}{7 \cdot 3!} + \frac{1}{11 \cdot 5!} - \frac{1}{15 \cdot 7!}}_{.3102681578}$ desired approx.

p.15 3: 10,

$$e^{-x}\tan x = \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\right)\left(x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots\right)$$

$$= x \qquad + \frac{x^3}{3} \qquad + \cdots$$

$$-x^2 \qquad + \cdots$$

$$+ \frac{x^3}{2} \qquad + \cdots$$

$$= x - x^2 + \frac{5}{6}x^3 + \cdots$$

7. $$\log(1+x) = \int \frac{dx}{1+x} = \int [1 - x + x^2 - x^3 + \cdots]\,dx$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$ (check both sides are 0 when $x=0$)

$$\sqrt{1+x} = (1+x)^{\frac{1}{2}}$$

$$= (1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + \tfrac{1}{16}x^3 - \tfrac{5}{128}x^4 + \cdots$$ (see problem 2 abov

$$x - x^2 + \frac{23}{24}x^3$$

$$1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + \tfrac{1}{16}x^3 \;\Big)\; x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots$$

$$x + \frac{x^2}{2} - \frac{x^3}{8} + \frac{x^4}{16} - \frac{5}{128}x^5 + \cdots$$

$$-x^2 + \frac{11}{24}x^3 - \frac{3}{16}x^4 + \frac{123}{640}x^5$$

$$-x^2 + \frac{1}{2}x^3 + \frac{1}{8}x^4 - \frac{x^5}{16}$$

$$\frac{23}{24}x^3 - \frac{1}{16}x^4 +$$

$$\frac{1}{3} + \frac{1}{8} = \frac{11}{24}$$

$$-\frac{3}{16} + \frac{1}{8} = \frac{-3+2}{16} = \frac{-1}{16}$$

$$\frac{1}{5} - \frac{5}{128} = \frac{128-5}{640}$$

$$= \frac{123}{640}$$

$$\frac{\log(1+x)}{(1+x)^{\frac{1}{2}}} = x - x^2 + \frac{23}{24}x^3 + \cdots$$

NOTE: AN EASIER CALCULATION:

$$\frac{\log(1+x)}{\sqrt{1+x}} = (1+x)^{-\frac{1}{2}}\log(1+x)$$

[it is easier to multiply power series].