Math 3302/2 Assignment 2

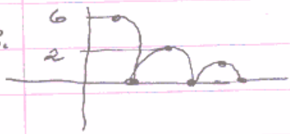
p. 104 12. BOOK'S NOTATION: The n-th term of a geometric series is ar^{n-1} $(n=1,2,\dots)$. We are given

$ar^3=-1$, $ar^6=\frac{1}{8}$, hence

$$r^3=\frac{ar^6}{ar^3}=\frac{-1}{8} \Rightarrow r=-\frac{1}{2}$$

$$ar^3=-1 \Rightarrow a\left(-\tfrac{1}{2}\right)^3=-1 \Rightarrow a=8$$

$$S=\frac{a}{1-r}=\frac{8}{1+\frac{1}{2}}=\frac{16}{3}.$$

13. 

$$D=6+\overbrace{2\cdot\left(\tfrac{1}{3}\cdot 6\right)+2\cdot\left(\left(\tfrac{1}{3}\right)^2\cdot 6\right)+\dots}^{\text{geometric series}}$$

$$a=2\cdot\tfrac{1}{3}\cdot 6=4 \qquad r=\tfrac{1}{3}$$

$$D=6+\frac{a}{1-r}=6+\frac{4}{1-\frac{1}{3}}=6+\frac{12}{3-1}=6+6=12$$

14. $$\sum_{n=1}^{\infty}\frac{n^2}{n+1}=\frac{1^2}{1+1}+\frac{2^2}{2+1}+\frac{3^2}{3+1}+\frac{4^2}{4+1}+\frac{5^2}{5+1}+\dots$$

$$=\frac{1}{2}+\frac{4}{3}+\frac{9}{4}+\frac{16}{5}+\frac{25}{6}+\dots$$

$\lim_{n\to\infty}\frac{n^2}{n+1}=\infty\neq 0$ hence diverges

19. $$\sum_{n=1}^{\infty}\frac{3n+2}{n^3}<\sum\frac{3n+2n}{n^3}=5\sum\frac{n}{n^3}=5\sum\frac{1}{n^2}<\infty$$

(COMPARISON TEST!)

$\Rightarrow$ here is how you can use the limit comparison test

compare with $\sum\frac{1}{n^2}$. We have

$$\frac{a_n}{b_n}=\frac{3n+2}{n^3}\Big/\frac{1}{n^2}=3+\frac{2}{n}\to 3<\infty$$

$p=2$ $\quad p=1$

hence $\sum b_n=\sum\frac{1}{n^2}<\infty \Rightarrow \sum a_n=\sum\frac{3n+2}{n^3}=3\sum\frac{1}{n^2}+2\sum\frac{1}{n^3}<\infty$

20. $$\sum_{n=1}^{\infty}\frac{n-2}{n^3}<\sum_{n=1}^{\infty}\frac{n}{n^3}=\sum\frac{1}{n^2}<\infty$$

III:2 $$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}=\sum_{1}^{\infty}\frac{1}{n^{1/2}}=\infty \qquad \left(p=\tfrac{1}{2}<1\right).$$

p. 111: 4 $\sum \frac{n+1}{n\sqrt{n}} = \underbrace{\sum \frac{1}{\sqrt{n}}}_{\infty} + \underbrace{\sum \frac{1}{n\sqrt{n}}}_{<\infty} = \infty$ DIVERGES

6. $\sum \frac{1}{n 2^n} < \underbrace{\sum \frac{1}{2^n}}_{\text{geometric series}} < \infty$ (easy comparison test)

12. $\sum_{n=1}^{\infty} \frac{2}{2^n+3} < \sum_{n=1}^{\infty} \frac{2}{2^n} = 2 \underbrace{\sum \frac{1}{2^n}}_{\text{geometric series } r=\frac{1}{2}<1} < \infty$

14. $\lim \frac{2^n}{1000} = \infty \Rightarrow \sum \frac{2^n}{1000} = \infty$

31. $\int_2^{\infty} \frac{dx}{x(\log x)^p} = \lim_{R\to\infty} \left(\int_2^R \frac{dx}{x(\log x)^p} \right)$

$= \lim_{R\to\infty} \left[\int \frac{dx}{x(\log x)^p} \right]_2^R$

$\underline{u = \log x}$ $du = \frac{dx}{x} \Rightarrow \int \frac{dx}{x(\log x)^p} = \int \frac{du}{u^p} = \int u^{-p}\,du$ $(p \neq 1)$

$(p \neq 1)$ $= \frac{u^{-p+1}}{-p+1} = \left(\frac{1}{1-p}\right) \frac{1}{(\log x)^{p-1}}$

$\underline{\underline{p>1}}$ $\int_2^{\infty} \frac{dx}{x(\log x)^p} \overset{*}{=} \lim_{R\to\infty} \left[\left(\frac{1}{1-p}\right) \frac{1}{(\log x)^{p-1}} \right]_2^R$

$= \lim_{R\to\infty} \left(\frac{1}{p-1}\right) \left[\frac{1}{(\log 2)^{p-1}} - \frac{1}{(\log R)^{p-1}} \right]$ $((-1)(-1)=+1)$

$= \frac{1}{p-1} \cdot \frac{1}{(\log 2)^{p-1}}$ (since $\log R \to \infty$ as $R \to \infty$)

$< \infty$

hence series converges for $p>1$. $\underline{\text{If } p<1}$

$\int_2^{\infty} \frac{dx}{x(\log x)^p} \overset{*}{=} \lim_{R\to\infty} \left(\frac{1}{1-p}\right) \left[(\log R)^{1-p} - (\log 2)^{1-p} \right] = \infty$

If $p=1$ $\int_2^{\infty} \frac{dx}{x(\log x)} = \lim_{R\to\infty} \int_2^R \frac{dx}{x\log x} = \lim_{R\to\infty} \left[\log\log x \right]_2^R$

$= \log\log R - \log\log 2 \to \infty$ $\int \frac{dx}{x\log x} = \log\log x$

hence $p \leq 1 \Rightarrow$ SERIES DIVERGES