Handout 7 A review of some things you should study for exam
(See also Assign. 1–4 review and answers for practice exam)

1(a) Definition of completion (N, θ) of a metric space M

b) Proof that completions are "essentially" unique (including definition)

c) Proof that completions exist

2.a) Definition of $\mathcal{E}([a,b])$ ($\subseteq \ell^\infty([a,b])$)

b) Proof that $C([a,b]) \subseteq \overline{\mathcal{E}([a,b])}^{\|\|_\infty}$

c) Construction of $I : \overline{\mathcal{E}([a,b])} \to [0,\infty)$

d) Definition of $\mathcal{R}([a,b])$ (Riemann integrable functions $f : [a,b] \to \mathbb{R}$) and proof that
$\overline{\mathcal{E}([a,b])} \subsetneq \mathcal{R}([a,b]) \subsetneq \ell^\infty([a,b])$

3. a) Definition of ring of sets $\mathcal{R}$ in a set Ω, σ-ring of sets, σ-algebra of sets, measure, $\mathcal{S}(\mathcal{R})$, ...

b) Proof that $\mathcal{R} = \{I_1 \sqcup \dots \sqcup I_m : I_k = [a_k, b_k)\}$ is a ring of sets

c) Definition of $\lambda : \mathcal{R} \to [0,\infty)$ and proof that it is a countably additive measure (see Handout #6)

4. Let $\mathcal{R}$ be a ring of sets in Ω such that $\Omega = \bigcup_{n=1}^{\infty} E_n$ for some sequence $E_n \in \mathcal{R}$, and

$\mu : \mathcal{R} \to [0,\infty]$ countably additive.

a) Define $\mu^+ : \mathcal{P}(\Omega) \to [0,\infty]$ and show it is countably subadditive: $S \subseteq \bigcup T_n \Rightarrow \mu^+(S) \le \sum \mu^+(T_n)$

b) Definition of $\mathcal{M}_\mu$ and proof that $\mathcal{S}(\mathcal{R}) \subseteq \mathcal{M}_\mu$ and fact that $\bar{\mu} = \mu^+|_{\mathcal{M}_\mu}$ is a countably additive extension of μ

c) Proof that if μ is σ-finite (def?) then extension to $\mathcal{M}_\mu$ is unique.

5. The Cantor set and its properties – see Handout 6.