Handout 4 Some Things you should know for Practice exam

1.a) Given set $S_\alpha \subseteq X$ $(\alpha \in I)$, the definition of $\prod_{\alpha \in I} S_\alpha$

b) The statement of the axiom of choice

c) The book's definition of card $X \leq$ card Y
and card $Y \geq$ card X

d) The proof that card $X \leq$ card $Y \Leftrightarrow$ card $Y \geq$ card X

2. Statement and proof of Schroder-Bernstein Theorem

3. Proofs that:

card $\mathbb{N}$ = card $\mathbb{Z}$ = card $\mathbb{Q}$ < card $\mathcal{P}(\mathbb{N})$ = card $\mathbb{R}$

4. Def.s: metric space, convergent sequence, Cauchy
4. sequence, complete metric space, normed vector space
Proof that $\mathbb{R}^m$ is complete ($\| \|_1$, $\| \|_2$, or $\| \|_\infty$).
Def.s: closed sets, open sets, proof: F closed $\Leftrightarrow F^c$ open.

5. Sequential def of compactness of (M, d)
Proof that M compact $\Rightarrow M$ bounded, and complete
$M \subseteq \mathbb{R}^m$ compact $\Leftrightarrow M$ bounded and closed

6. Given M_n $(n \in \mathbb{N})$ metric spaces, $d_n \leq 1$,
a) The def. of a metric d on $\prod M_n$
b) The proof that M_n compact $\Rightarrow \prod M_n$ compact.

7. Continuity and uniform continuity (with examples)

8. $f: M \to N$ cont., M compact $\Rightarrow f$ unif cont

9: $A \subseteq M$, $f: A \to N$ unif cont, N complete $\Rightarrow$
f has a unique extension to $\bar{f}: \bar{A} \to N$

10. Proof that $\ell^\infty(\mathbb{R})$ is complete
Proof that if M any metric space, $\exists$ isom $\theta: M \to \ell^\infty(\mathbb{R})$

11. Proof that if E and F are normed spaces and $\varphi: E \longrightarrow F$ linear then following are equivalent:

(1) φ is continuous at a point

(2) φ is continuous at every point

(3) φ is bounded [know the definition]