Assignment 3 : PROVE YOUR ASSERTIONS

1. Show that if $a \le x \le b$, then

   a) $\mathbb{1}_{\{a\}} \notin \overline{\mathcal{E}([a,b])}^{\|\cdot\|_\infty}$

   b) $\mathbb{1}_{\{a\}} \in \mathcal{R}([a,b])$

   c) give an example of a function $f \in \ell^\infty([a,b]) \setminus \mathcal{R}([a,b])$

2. Show that the collections

$$\mathcal{F}_1 = \{[a,b] : a < b\} \qquad a, b \in \mathbb{R}$$
$$\mathcal{F}_2 = \{[a,b) : a < b\} \qquad \text{"}$$
$$\mathcal{F}_3 = \{(a,b) : a < b\} \qquad \text{"}$$

generate the same σ-algebras of sets, i.e.,

$$\mathcal{S}(\mathcal{F}_1) = \mathcal{S}(\mathcal{F}_2) = \mathcal{S}(\mathcal{F}_3)$$

3. Compare $\mathcal{R}(\mathcal{F}_1)$ and $\mathcal{R}(\mathcal{F}_2)$ (the rings generated by $\mathcal{F}_1$ and $\mathcal{F}_2$).

4. Show that if $\mathcal{R}$ is a ring of subsets of a set X, then $\mathcal{R}$ is a ring in the <u>algebraic</u> sense under the operations

$$E + F = (E \setminus F) \cup (F \setminus E) \qquad (\text{also denoted } A \Delta B)$$
$$E \cdot F = E \cap F$$