Assignment 1 due Monday

1. Suppose that S and T are sets. Prove that
$$\text{card}(S \times T) = \text{card}(T \times S).$$

2. Prove that the following sets are countable:
   a) the set of all real numbers with a distinct decimal expansions (e.g. .500... and .4999...)
   b) the set of all finite subsets of $\mathbb{N}$.
   Hint: you might want to use the "fundamental theorem of arithmetic" $n = 2^{a_1} \cdots p_m^{a_m}$ (unique rep)

3. Let $\mathcal{X}$ be the set of all functions $f: \mathbb{N} \to \{0, 1\}$.
Prove that $\text{card}\, \mathcal{X} = \text{card}\, \mathbb{R}$.

4. Show that $\text{card}\,[0,1] = \text{card}\,(0,1)$.
(Hint: you can use the Cantor-Bernstein Theorem).