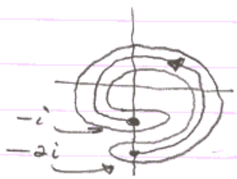
Math 132 <u>Practice Exam</u>

1 a) Show that one can define $\log z$ in the indicated region



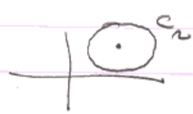
b) Use the Cauchy-Riemann equations to show $\log z$ is analytic in that region and to show that

$$\frac{d}{dz}\log z = \frac{1}{z}$$

c) Calculate $\int_\gamma \frac{dz}{z}$ for the indicated curve

2. Use the formula

$$f'(z) = \frac{1}{2\pi i}\int_{C_r} \frac{f(s)}{(s-z)^2}\,ds$$



to prove that if f is analytic on $\mathbb{C}$, and f is bounded, then f is constant.

3. Evaluate by contour integration

$$\int_{-\infty}^{\infty} \frac{\cos^2 x}{1+x^2}\,dx$$

[Include the proof that a particular contour integral goes to zero as $R \to \infty$]

4. Find a fractional linear transformation that maps $(0, 1, \infty)$ to $(-i, \infty, 1)$

5. Find the residues of

$$\frac{e^z}{z^2(\pi - z)}$$ at each of its poles

6. Find the principal part of

$$z^2 \sin\left(\frac{1}{z}\right) \qquad \text{at } z=0.$$

7. Show that if u is harmonic on $\mathbb{C}$ and it is bounded below, then it is constant.

8. Give examples:

a) $f_n : [0,1] \to \mathbb{R}$ $f_n \to f$ pointwise, but $\int_0^1 f_n(x)\,dx \not\to \int_0^1 f(x)\,dx$

b) $f_n : [0,1] \to \mathbb{R}$, f_n continuous, $f_n \to f$ pointwise, but f not continuous.

c) $f_n : [0,1] \to \mathbb{R}$, f_n continuously differentiable, $f_n \to f$ uniformly, but f not differentiable (at some point)

9. Sketch the proof that $f_n : G \to \mathbb{C}$ analytic $f_n \to f$ uniformly $\Rightarrow$ f analytic and $f_n^{(k)} \to f^{(k)}$

10. Find $\int_{\gamma_1} \bar{z}\,dz$ and $\int_{\gamma_2} \bar{z}\,dz$ for the indicate curves