

Practice Midterm Exam (Math 132) NOTE: This is longer than an hour exam.

1. Determine the following ...

a)  $\sqrt[3]{1-i}$

e)  $\log(1+i)$

c)  $\cos(1+i)$

d)  $\operatorname{Im} e^{2+i}$

2. Sketch the image of  $y = x+1$  under the map  $f(z) = z^2$ .

3. Prove that for any complex numbers  $z, w$

$$\cos(z+w) = \cos z \cos w - \sin z \sin w$$

4. Let  $f(z) = e^{3z}$ . Use the Cauchy Riemann equations to

a) show that  $f(z)$  is analytic on  $\mathbb{C}$

b) find the derivative  $f'(z)$

5. Show that  $x^2 - y^2 + x$  is harmonic and find its harmonic conjugate.

6. Find a fractional linear transformation

$$f(z) = \frac{az+b}{cz+d}$$
 that maps the circle

$|z|=1$  onto the real line. Where does  $f$

map the interior of the circle?

7. Define  $f(z) = |z| + i \operatorname{Arg} z$ . Is this analytic?

Prove your assertion (Hint: Write  $|z|$  and  $\operatorname{Arg} z$  as functions of  $x$  and  $y$ )