Completion of Problem 2 on practice exa

From the answer sheet, we have shown that

$$\int_{-\infty}^{\infty} \frac{e^{i2x}}{1+x^2}\,dx = \frac{\pi}{e^2}$$

Since $e^{i2x} = \cos 2x + i \sin 2x$

$$\int_{-\infty}^{\infty} \frac{\cos 2x}{1+x^2}\,dx = \mathrm{Re} \int_{-\infty}^{\infty} \frac{e^{i2x}}{1+x^2}\,dx = \mathrm{Re}\,\frac{\pi}{e^2} = \frac{\pi}{e^2}$$

Finally,

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1-\cos^2 x)$$

$$= 2\cos^2 x - 1$$

hence $\cos^2 x = \dfrac{\cos 2x + 1}{2}$

$$\int_{-\infty}^{\infty} \frac{\cos^2 x}{1+x^2}\,dx = \frac{1}{2}\int_{-\infty}^{\infty} \frac{\cos 2x}{1+x^2}\,dx + \frac{1}{2}\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$$

$$= \frac{\pi}{2e^2} + \frac{\pi}{2}$$

- we have used $\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \pi$. On the exam I would give you that value (or in a seperate problem ask you to prove it).