Solutions to Assignment 1 WATCH FOR STUPID ERRORS!

1. For all $n \in \mathbb{N}$, tell the person in room n to go to room $n+1$. You can then use room 1 since it is empty.

2. Let $t_0 = 11$; $t_1 = 11{:}30$, $t_2 = 11{:}45$, ... $t_n = 11{:}t_n'$ where $t_n' = 60(1-2^{-n})$.

a) For any $n \in \mathbb{N}$, ~~the~~ the n-th ball will be removed at time t_n. So no balls left at 12.

b) At any time t_n, the balls $2, 3, 4, \ldots, 100, 102, \ldots$ will remain in the urn. So ∞ many balls at 12.

1.8 For any number z we know that $z^2 \geq 0$. Let $z = x - y$. Then $0 \leq z^2 = x^2 - 2xy + y^2 \Rightarrow 2xy \leq x^2 + y^2$

1.9 $|x_1 + x_2 + x_3| = |(x_1 + x_2) + x_3| \leq |x_1 + x_2| + |x_3| \leq |x_1| + |x_2| + |x_3|$

↳ we already proved this

1.10 $|x| = |x - y + y| \leq |x - y| + |y|$ (triangle ineq)

hence $|x| - |y| \leq |x - y|$. Interchanging x and y

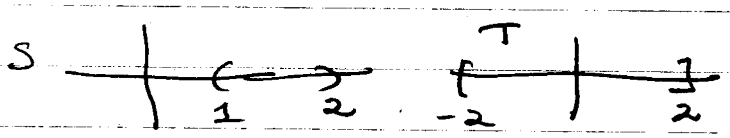
$|y| - |x| \leq |y - x| = |x - y|$

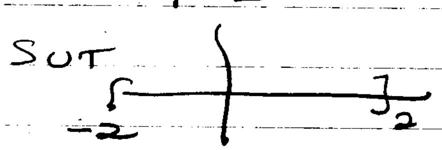
hence

$-|x-y| \leq |x| - |y| \leq |x - y|$

and thus $\big||x| - |y|\big| \leq |x - y|$ [we showed $|a| \leq c \iff -c \leq a \leq c$]

2.1

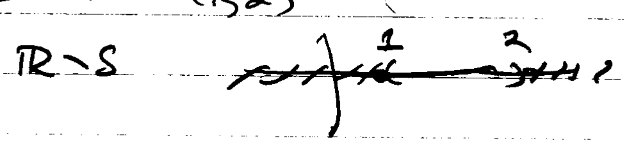




$S \cup T = [-2, 2]$

$(S \subseteq T \Rightarrow S \cup T = T)$

$S \cap T = (1, 2)$



$\mathbb{R} \setminus S = (-\infty, 1] \cup [2, \infty)$

$T \setminus S = [-2, 1] \cup \{2\}$. $\mathbb{R} \setminus (T \setminus S) = (-\infty, -2) \cup (1, 2) \cup (2, \infty)$