Math 32b F05 Practice Final 2005

spherical co-ordinates: $x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$, $z = \rho \cos\phi$, $\frac{\partial(x,y,z)}{\partial(\rho,\theta,\phi)} = \rho^2 \sin\phi$

1. Evaluate $\int_0^1 \int_{y=x}^1 \sin(y^2) dy dx$. (Hint: you should first change the order of integration.)

2. A triangular plate has the vertices (0,0), (2,1), and (0,1). Its density at (x, y) is y. Find the center of mass of the plate.

3. Let P be the parallelogram spanned by the vertices $(0,0)$, $(2,0)$, $(1,2)$, $(3,2)$. Use the change of variables $x = u + v$, $y = 2v$ to evaluate $\int\int_P (x + y) dx\, dy$.

4. a)Sketch the level curves of the function $f(x, y) = \arctan(y/x)$ (Hint: this is the polar angle θ of the point (x, y)), and sketch the vector field ∇f on this same picture.

   b) Calculate ∇f .

5. Find $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y) = (y^2 + 2xy)\vec{\imath} + (2xy + x^2)\vec{\jmath}$ where C is an arbitrary curve from $(1, 1)$ to $(2, 3)$.

6. a) Give a parametric equation for the plane through the points $P = (0, 1, 2)$, $Q = (1, 2, 0)$, $R = (1, 1, 1)$. Hint the easiest parametrization is $\vec{r}(u, v) = u\vec{P} + v\vec{Q} + (1 - u - v)\vec{R}$.

   b) Find $\int\int_T (x + y - z) dS$ where T is the triangle with vertices P, Q, R, and $0 \le u \le 1, 0 \le v \le 1$, $u + v \le 1$.

7. Use Green's theorem to compute $\int \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = 2yx\vec{\imath} + 3x^2\vec{\jmath}$ and C is the circle with center $(0, 0)$ and radius 2.

8. Use Stoke's theorem to evaluate $\int_S \text{curl}\vec{F} \cdot d\vec{S}$, where $\vec{F} = (x^2 + y)\vec{\imath} + 3xy\vec{\jmath} + (2xz + z^2)\vec{k}$ and S is the surface of the paraboloid $z = 4 - x^2 - y^2$ above the xy plane with normal vector pointing up.

9. Find the area spanned by the spiral ramp

$$\vec{r}(u, v) = u \cos v\vec{\imath} + u \sin v\vec{\jmath} + v\vec{k} \quad 0 \le u \le 1, 0 \le v \le 3\pi$$

10. Use the divergence theorem to evaluate $\int\int_S \vec{F} \cdot d\vec{S}$ where S is the sphere with center 0 and radius a, and $\vec{F} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$.