

Math 32b Practice second hour exam.

1. Let B be the region described by

$$x \geq 0, y \geq 0, z \geq 0, \frac{1}{2} \leq x^2 + y^2 + z^2 \leq 1.$$

- a) Sketch this region, and describe it in terms of spherical co-ordinates ρ, θ, ϕ .
- b) Use spherical co-ordinates to evaluate

$$\int \int \int_B \sqrt{x^2 + y^2 + z^2} dV.$$

Hint: $\left| \frac{\partial(x,y,z)}{\partial(\rho,\theta,\phi)} \right| = \rho \sin^2 \phi$.

2. Let $(x, y) = T(u, v)$ be defined by $x = u^2 - v^2, y = 2uv$.

- a) If R is the region described by $1 \leq u^2 + v^2 \leq 4, u \geq 0, v \geq 0$, sketch the region $E = T(R)$.
- b) Use this change of variable to evaluate the integral

$$\int \int_D \frac{dx dy}{\sqrt{x^2 + y^2}}.$$

Include a calculation of the Jacobian determinant.

3. a) Sketch the vector field $\vec{F}(x, y) = y\vec{i} - x\vec{j}$.

b) Determine $\int \vec{F} \cdot d\vec{r}$ where C is the unit circle at the origin traversed in the counterclockwise direction from the positive X axis to the negative X axis.

4. Determine the following line integrals:

a) $\int_{\Gamma} y dx + x^2 dy$, where Γ is given by $x = t, y = t^2, -1 \leq t \leq 2$. Include a sketch of the curve.

b) $\int_T y dx + x^2 dy$ where T is the triangle determined by the points $(0, 0), (1, 2), (-1, 3)$ traversed in the clockwise direction (include a sketch of the curve with arrows indicating the direction of the motion).