Math 131a/2 W05 First Hour Practice Exam

1. Prove that for any sets $A, B \subseteq X$,

$$(A \cup B)^c = A^c \cap B^c.$$

2. Define:

a) $f : X \to Y$ is a one-to-one function

b) $f : X \to Y$ is an onto function

c) $f : X \to Y$ is a one-to-one correspondence

d) X and Y have the same cardinality

e) X is a finite set

f) X is countable.

2. Prove that for any real numbers x and y, $||x| - |y|| \leq |x - y|$.

3. Prove that if $f : \mathbb{N} \to Y$ is an onto function, then Y is countable.

4. Prove that $\mathbb{N} \times \mathbb{N}$ is countable.

5. Prove that 24 does not have an integer cube root.

6. Prove that if S is a subset of $\mathbb{N}$, then S must be countable. Carefully explain what property of $\mathbb{N}$ that you are using.