Math 131a Handout #4
Assignment 3

1. p. 50:5, 7
2. p. 54: 1, 2, 5
3. p. 59: 1, 2, 6, 7
4. p. 79: 3,5,6,7,8
5. p. 86:1,2,3,6,7,9,10

Our **completeness axiom** (see handout #3): If S is a non-empty subset of $\mathbb{R}$, and S is bounded above (i.e., $S \leq b$ for some b), then S has a least upper bound $b_0 = \sup S$. (You fomulate the corresponding result for non-empty sets that are bounded below).

Here are theorems about sequences and their limts that you should be able to prove (including the relevant definitions):

- If x_n is a convergent sequence, then it must be bounded.
- If $x_n \to L$ and $x_n \neq 0$ and $L \neq 0$, then there is a constant $c > 0$ such that $|x_n| \geq c$ for all n.
- If $x_n \to L$ and for all n, $x_n \geq 0$, then $L \geq 0$.
- The usual limit theorems (such as $x_n \to L$ and $y_n \to M$ implies $x_n + y_n \to L + M$)
- If x_n is an increasing sequence, and $x_n \leq b$, then $x_n \to b_0 = \sup \{x_n\}$. (You should be able to state and prove the corresponding result for decreasing sequences).
- If $\emptyset \neq S \subseteq \mathbb{R}$ and $b_0 = \sup S$, then there is a sequence $x_n \in S$ such that $x_n \to b_0$. (You should be able to state and prove the corresponding result for the infimum).
- If x_n is a convergent sequence, then it must be Cauchy.
- If x_n is a Cauchy sequence, then it must be bounded.
- If $x_n \to L$, then for any subsequence x_{n_k}, $x_{n_k} \to L$.
- Every sequence has a monotonic subsequence.
- If x_n is a Cauchy sequence and a subsequence $x_{n_k} \to L$, then $x_n \to L$.
- If x_n is a Cauchy sequence, then it must converge.
- If x_n is a bounded sequence, then it has a convergent subsequence. [This is called the **Balzano-Weierstrass Theorem** – see page 57 of the text.]