(1) Put a relation $E$ on the integers by $x \sim y$ if $x - y$ is divisible by 2. Show that $E$ is an equivalence relation. What are all the elements related to 0? What about all the elements related to 1?

(2) Let $X$ be a set and $f : X \to \mathbb{R}$ a function. Consider the relation on $X$ given by $a \sim b$ if $f(a) \geq f(b)$.
   (a) Show that the relation is transitive and reflexive.
   (b) What condition on $f$ is necessary for this relation to be anti-symmetric?
   (c) Describe how by choosing $X$ and $f$ appropriately this relation can give relations on people such as “$a$ is related to $b$ if $a$ is wealthier than $b$”, or “$c$ is related to $d$ if $c$ is older than $d$”.

(3) Put a relation $Q$ on $\mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$ by $(a, b) \sim (c, d)$ if $ad = bc$. Show that $Q$ is an equivalence relation. What mathematical structure does this remind you of?

(4) How many symmetric operations are there on an $n$ element set?

(5) Consider the integers between 5 and 200, inclusive.
   (a) How many are even? How many are odd?
   (b) How many are divisible by 5?
   (c) How many do not contain the digit 0?
   (d) How many contain distinct digits?
   (e) How many have digits in a strictly increasing order?
   (f) How many are greater than 101 and do not contain the digit 6?

(6) A family of five consisting of a mother, a father, and 3 (distinct!) children are going to stand in a line to take a photo.
   (a) In how many ways can they do this?
   (b) In how many ways can they do this if the mother and father are going to stand next to each other?
(7) How many terms are there in the expansion of 
\[(x + y)(a + b + c)(e + f + g)(h + i)\]?