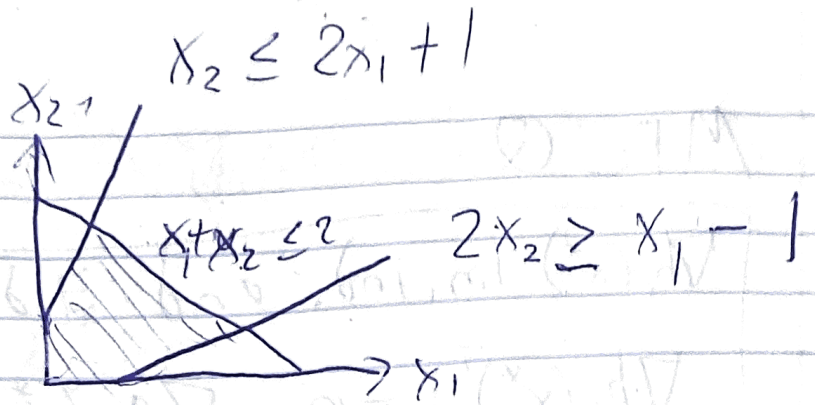

Min $-x_1 - x_2$
s.t. $-2x_1 + x_2 \leq 1$
$-x_1 + 2x_2 \geq -1 \Rightarrow$
$x_1 + x_2 \leq 2$
$x_1, x_2 \geq 0$

Min $-x_1 - x_2$
s.t. $-2x_1 + x_2 + s_1 = 1$
$+x_1 - 2x_2 + s_2 = 1$
$x_1 + x_2 + s_3 = 2$
$x_1, x_2, s_1, s_2, s_3 \geq 0$

Simplex Tableaux

x_1	x_2	s_1	s_2	s_3	b
-2	1	1	0	0	1
1	-2	0	1	0	1
1	1	0	0	1	2
-1	-1	0	0	0	0

How to read Simplex Tableaux

1. Is it canonical with respect to a basis?
   If there are columns which form the identity, then yes.
2. What are basis variables?
   These are the variables with columns forming identity
3. What are the values of basis variables?
   This is the value on right hand side
4. What are nonbasis vars? The remaining
5. What are their values? 0!
6. What is the current function value?
   negative of the lower right hand side
7. Is solution feasible?

How to perform Simplex algorithm

1. Find non-basis var with most negative cost
2. Compute ratios of b column to x column
3. Pick the row with smallest nonnegative ratio.
4. Perform elimination on x column until column has 1 in this row and 0 elsewhere.
5. Repeat until no negative costs.

First Step

1. We pick x_1
2. Ratios are

$$\begin{bmatrix} 1/-2 \\ 1/1 \\ 2/1 \end{bmatrix}$$

← negative ignore
← smallest positive

3. 2nd row chosen
4. After elimination

x_1	x_2	s_1	s_2	s_3	b
0	-3	1	2	0	3
1	-2	0	1	0	1
0	3	0	-1	1	1
0	-3	0	1	0	1

Basis: $s_1 = 3$, $x_1 = 1$, $s_3 = 1$
Non basis: $x_2 = 0$, $s_2 = 0$
Function value: -1

2nd step

1. Pick x_2
2. Ratios

$$\begin{bmatrix} 3/-3 \\ 1/-2 \\ 1/3 \end{bmatrix}$$

← least positive

3. Pick row 3
4. After elimination

x_1	x_2	s_3	s_2	s_3	b
0	0	1	1	1	4
1	0	0	1/3	2/3	5/3
0	1	0	-1/3	1/3	1/3
0	0	0	0	1	2

Basis: $s_3 = 4$ $x_1 = 5/3$ $x_2 = 1/3$
Nonbasis: $s_1 = s_2 = 0$

Value: -2

Phase I: If we don't have basic solution,

Convert to Associated Problem

$$\min\ -x_1 - 2x_2$$
$$\text{s.t.}\ \ x_1 + x_2 = 1$$
$$x_1 + 2x_3 = 2$$
$$x_2 + 2x_3 = 2$$
$$x_1, x_2, x_3 \geq 0$$

$$\min\ -x_1 - 2y_2$$
$$\text{s.t.}\ \ x_1 + x_2 + y_1 = 1$$
$$x_1 + 2x_3 + y_2 = 2$$
$$x_1, x_2, x_3, y_1, y_2 \geq 0$$

x_1	x_2	x_3	y_1	y_2	
1	1	0	1	0	1
1	0	2	0	1	2
0	0	0	1	1	0

↓ Convert to canonical form

x_1	x_2	x_3	s_1	s_2	
1	1	0	1	0	1
1	0	2	0	1	2
-2	-1	-2	0	0	-3

1. Pick x_1
2. ratios $\begin{bmatrix} 1/1 \\ 2/1 \end{bmatrix}$ ← smallest
3. Eliminate

x_1	x_2	x_3			
1	1	0	1	0	1
0	-1	2	-1	1	1
0	1	-2	2	0	-1

Basis x_1, x_3

1. Pick x_3
2. ratios $\begin{bmatrix} 1/0 \\ 1/2 \end{bmatrix}$ ← smallest
3. Pick row 2
4. Eliminate

1	1	0	1	0	1
0	-1/2	1	-1/2	1/2	1/2
0	0	0	1	1	0

Do simplex method again after dropping y_1, y_2 columns and using old costs

Last step

x_1	x_2	x_3	
1	1	0	1
0	-1/2	1	1/2
-1	-2	0	0

$\Rightarrow$

1	1	0	1
0	-1/2	1	1/2
0	-1	0	1

Now perform simplex method