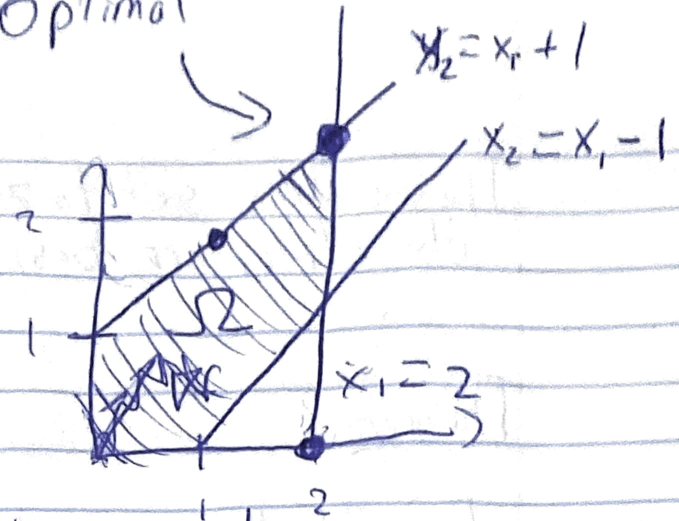
(2,3) optimal



$\max \; x_1 + 2x_2$

s.t.

$-x_1 + x_2 \le 1$

$-x_1 + x_2 \ge -1 \iff x_1 - x_2 \le 1$

$x_1 \le 2$

Standard Form

Introduce $s_1, s_2, s_3 \ge 0$

$\min \; -x_1 - 2x_2$

$$\begin{aligned} \text{s.t.} \quad -x_1 + x_2 + s_1 &= 1 \\ x_1 - x_2 + s_2 &= 1 \\ x_1 + s_3 &= 2 \\ x_1, x_2, s_1, s_2, s_3 &\ge 0 \end{aligned}$$

~~Theorem~~

Def: a point x in a set X is an extreme point of X if x cannot be written as $x = \frac{y+z}{2}$ for $y, z \in X$

If X is a polyhedron, the extreme points of X are vertices.

~~Thm.~~ Def: Given an LP in standard form, $\begin{cases} \min c^T x \\ \text{s.t. } Ax = b \;]\; m \text{ equ} \\ x \ge 0 \;]\; n \text{ ineq} \end{cases}$

a point x is feasible if $Ax = b$, $x \ge 0$,

a point x is basic if it is given by fixing $n - m$ variables to 0 and solving $Ax = b$ for the remainder

The variables set to zero are nonbasic
the others are basic variable (may $=0$)

<u>Theorem</u>:

x is a Feasible Basic point ~~s~~ iff x is extreme point of the feasible region iff x is a vertex of feasible region

Examples:

Choose $x_1, x_2 = 0$, remaining system gives $s_1 = 1$
$s_2 = 1$
$s_3 = 2$

This is feasible as all $x_1, x_2, s_1, s_2, s_3 \geq 0$

Choose $x_2 = 0$, $s_3 = 0$, remaining system gives

$$-x_1 + s_1 = 1$$
$$+x_1 + s_2 = 1$$
$$x_1 = 2$$

$\Rightarrow x_1 = 2, x_2 = 0, s_1 = 2, s_2 = -1, s_3 = 0$

Not feasible since $s_2 \leq 0$

A basic solution is feasible if and only if all basic variables are ≥ 0

Simplex Method

Systematic way of determining feasible basic solutions

Tableaux

x_1	x_2	s_1	s_2	s_3	b
-1	1	1	0	0	1
1	-1	0	1	0	1
1	0	0	0	1	2
-1	-2	0	0	0	0

Values of basic variables

feasible if all ≥ 0

How much objective changes if we increase x_1, x_2

will store negative of optimal value

Observe, it is very easy to determine s_1, s_2, s_3 if we choose x_1, x_2 to be nonbasic as the columns are the identity.

This tableaux is canonical wrt variables s_1, s_2, s_3

Given a tableaux, we identify basis variables as their columns form a permutation of identity matrix

Simplex Method:

1. Identify nonbasic variable x with ~~largest~~ most negative cost coefficient
2. Identify which row limits the size of x the most
3. Identify ~~2.~~ The basic variable y with a 1 in this row
4. Move x into basis and y out of basis by Gaussian Elimination
5. Repeat until nonegative cost coefficients

x_1	x_2	s_1	s_2	s_3	b
-1	1	1	0	0	1
0	0	1	1	0	2
1	0	0	0	1	2
-3	0	2	0	0	2

0	1	1	0	1	3
0	0	1	1	0	2
1	0	0	0	1	2
0	0	2	0	3	8

Done as all cost coefficients negative

Suppose I drop 1st constraint

What is optimal value?
infinite

No minimizer

1	-1	1	0	0	1
1	0	0	0	1	2
-1	-2	0	0	0	0

Can increase x_2 without limit
so problem is unbounded