Discussion 7 Prep

Linear Programming

Setup:

$$\min c^T x$$
$$\text{s.t. } Ax = b$$
$$x \geq 0$$

$c \in \mathbb{R}^n$

$A \in \mathbb{R}^{m \times n}$ $b \in \mathbb{R}^m$

↑ matrix vector Form

↓ component Form

$$\min \; c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n = b_1$$

$$\vdots$$

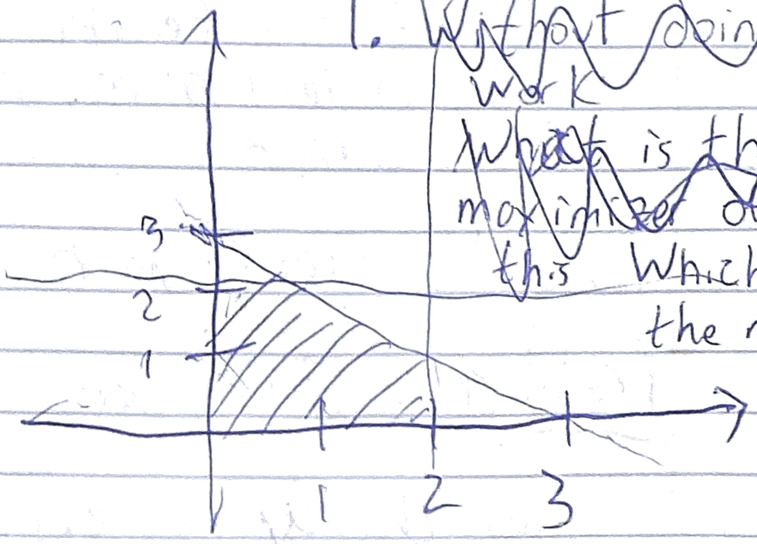
$$a_{m1} + a_{m2} x_2 + \dots + a_{mn} x_n = b_m$$

$$x_1, \dots, x_n \geq 0$$

Example Problem:

$$\max x_1 + 2x_2$$
s.t.
$$x_1 \leq 2$$
$$x_2 \leq 2$$
$$x_1 + x_2 \leq 3$$

$$x_1, x_2 \geq 0$$



~~1. Without doing any work~~
~~What is the maximizer of this~~

Which point is the maximum

How to find maximizer

~~Fact 1: If a minimizer/maximizer exists it occurs~~

Fact 1: The feasible region of an LP is a (possibly unbounded) polyhedron

Fact 2: If a minimizer/maximizer exists, it occurs at a vertex of the polyhedron

Fact 3: Vertices correspond to setting enforcing n of the constraints to hold at equality.

Question: Does every choice of n constraints correspond to a feasible point?

Basic Algorithm for solving LPs

~~Check every subset of n constraints,~~

For each subset of n constraints,

1. Solve system of n equations
2. If it is feasible, check if better than current min, if so update current min

Return current min

Problem: There are $\binom{m+n}{n}$ choices which grows exponentially in m, n

Need a way to systematically walk from vertex to vertex to find a minimizer

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Standard Form LPs $\quad m \leq n$

$$\min \; c^T x \qquad c \in \mathbb{R}^n$$
$$A \in \mathbb{R}^{m \times n} \quad x \in \mathbb{R}^n$$
$$\text{s.t. } Ax \leq b$$
$$x \geq 0$$

Already chose m constraints to be equality

Just need to choose $m-n$ components of x to equal zero

How to convert to standard form?

If have $x_1 \leq 0$ Let $\tilde{x}_1 = -x_1$,
$$\tilde{x}_1 \geq 0$$

If have $a_1x_1 + a_2x_2 + \dots + a_nx_n \leq b$

Let variable s be the "slack"
$$s = b - a_1x_1 + a_2x_2 + \dots + a_nx_n$$

$$a_1x_1 + a_2x_2 + \dots + a_nx_n + s = b, \quad s \geq 0$$

If no constraint on x, $(x \in \mathbb{R})$

Can write $x = x^+ - x^-$, $x^+, x^- \geq 0$

$$\min \; -x_1 - 2x_2$$

$$\begin{aligned} \text{s.t. } x_1 + s_1 \qquad\qquad &= 2 \\ x_2 + s_2 \qquad &= 2 \\ x_1 + x_2 + \qquad s_3 &= 3 \end{aligned}$$

$$x_1, x_2, s_1, s_2, s_3 \geq 0$$